DIGITALNA ELEKTRONIKA

Z A D A C I

I. BROJNI SISTEMI I KODOVI

1. Konvertovati sledeće decimalne brojeve u binarne:

a) **260** b) **0.0625** c) **3,166**

Rešenje:

a) $\underline{260} : 2 = \underline{130} : 2 = \underline{65} : 2 = \underline{32} : 2 = \underline{16} : 2 = \underline{8} : 2 = \underline{4} : 2 = \underline{2} : 2 = \underline{1} : 2 = 0$

0	0	1	0	0	0	0	0	1

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$\mathbf{260_{(10)} = 100000100_{(2)}}$

b)

0,0625	x 2 = 0,125	0	
0,125	x 2 = 0,25	0 ↓	$\mathbf{0{,}0625_{(10)} = 0{,}0001_{(2)}}$
0,25	x 2 = 0,5	0	
0,5	x 2 = 1,0	1	

c) $3{,}166_{(10)} = 3_{(10)} + 0{,}166_{(10)}$

$3_{(10)} = 11_{(2)}$

0,166	x 2 = 0,332	0
0,332	x 2 = 0,664	0 ↓
0,664	x 2 = 1,328	1
0,328	x 2 = 0,656	0
0,656	x 2 = 1,312	1
0,312	x 2 = 0,624	0
0,624	x 2 = 1,248	1

$0{,}166_{(10)} = 0{,}0010101..._{(2)}$

$\mathbf{3{,}166_{(10)} = 11{,}0010101_{(2)}}$

2. Konvertovati sledeće decimalne brojeve u binarne:

a) **125,875** b) **500,4375** c) **31,6**

Rešenje:

a) $\mathbf{125{,}875_{10} = 1111101{,}111_{2}}$ **b)** $\mathbf{500{,}4375_{10} = 111110100{,}0111_{2}}$

c) $\mathbf{31{,}6_{10} = 11111{,}1001100110..._{2}}$

3. Zadat je broj u binarnom brojnom sistemu: **101101011100.**
Napisati zadati broj u oktalnom i heksadecimalnom brojnom sistemu.

a) $\mathbf{101101011100_{2}} = 101\ 101\ 011\ 100_{2} = \mathbf{5534_{8}}$

b) $\mathbf{101101011100_{2}} = 1011\ 0101\ 1100_{2} = \mathbf{B5C_{16}}$

4. Predstaviti decimalni broj **847**

a) u kodu BCD 8421 b) u kodu BCDXS3 (kod "vise3")

c) u kodu BCD 2421 d) u Grejovom kodu

Rešenje:

a) $\mathbf{847_{10} = 100001000111_{2}}$ b) $\mathbf{847_{10} = 101101111010_{BCDXS3}}$

c) $\mathbf{847_{10} = 111001001101_{BCD2421}}$ d) $\mathbf{847_{10} = 110001100100_{BCDGrej}}$

5. Dati su BCD brojevi

a) **1100110,00111** b) **110100,001**

Napisati ove brojeve u dekadnom brojnom sistemu

Rešenje:

a) $\mathbf{1100110{,}00111_{BCD}}$ = 0110 0110 , 0011 1000 = $\mathbf{66{,}38_{10}}$

b) $\mathbf{110100{,}001_{BCD}}$ = 0011 0100 , 0010 = $\mathbf{34{,}2_{10}}$

6. Dati brojevi napisani su u BC oktalnom kodu:

a) **10011101** b) **101,11011101**

Rešenje:

a) $\mathbf{10011101_{BCO}}$ = 010 011 101 = $\mathbf{235_{8}}$

b) $\mathbf{101{,}11011101_{BCO}}$ = 101 , 110 111 010 = $\mathbf{5{,}672_{8}}$

7. Napisati sledeće heksadecimalne brojeve u binarno – kodovanom heksadecimalnom kodu (BCH)

a) **2AC** b) **F5,2A**

Rešenje:

a) $\mathbf{2AC_{16}}$ = 0010 1010 1100 = $\mathbf{1010101100_{BCH}}$

b) $\mathbf{F5{,}2A_{16} = 11110101\ {,}00101010_{BCH}}$

8. Pomnoziti brojeve **21** i **3,375** konvertujući ih u binarni brojni sistem. Dobijeni binarni broj ponovo konvertovati u decimalni.

Rešenje:

$21_{10} = 10101_2$ $3_{10} = 11_2$

0,375 x 2 = 0,75 0

0,75 x 2 = 1,5 1 ↓

0,5 x 2 = 1,0 1

$\mathbf{3{,}375_{10} = 11{,}011_2}$

```
11,011  x  10101
      11011
    110110
110110
1000110,111
```

$1000110{,}111_2$ $= 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 +1 \times 2^2 +1 \times 2^1 +0 \times 2^0 + 1 \times 2^{-1} +1 \times 2^{-2} + 1 \times 2^{-3} = 1 \times 64 +1 \times 4 + 1 \times 2 +1 \times 0.5 + 1 \times 0{,}25 + 1 \times 0{,}125 = 64 +4 +2 + 0{,}875 =$ **70,875**.

9. Pomnožiti brojeve **50,25** i **6** u binarnom sistemu; zatim proizvodu dodati broj **1001001,01.** Dobijeni binarni broj**:**

- **a)** konvertovati u decimalni
- **b)** predstaviti u oktalnom(OCT) i heksadecimalnom (HD) brojnom sistemu
- **c)** Napisati njihov proizvod u kodu BCD „više 3" (BCDXS3)

Rešenje:

$\underline{50} : 2 = \underline{25} : 2 = \underline{12} : 2 = \underline{6} : 2 = \underline{3} : 2 = \underline{1} : 2 = 0$ **$50_{10} = 110010_2$**

0 1 0 0 1 1

ost. ←

$0{,}25 \times 2 = 0{,}5$ 0 ↓ **$0{,}25_{10} = 0{,}01_2$**

$0{,}5 \times 2 = 1{,}0$ 1

$50{,}25_{10} = 110010{,}01_2$ **$6_{10} = 110_2$**

```
110010,01 x 110
   00000000                 100101101,10
  11001001               +    1001001,01
       11001001                101110110,11
     100101101,10
```

a) **$101110110{,}11_2$** $= 1\times2^8 + 0 \times 2^7 + 1\times2^6 + 1\times2^5 + 1\times2^4 + 0\times2^3 + 1\times2^2 + 1\times2^1 + 0\times2^0 + 1\times2^{-1} + 1\times2^{-2} = 256 + 64 + 32 + 16 + 4 + 2 + 0{,}5 + 0{,}25 =$ **$374{,}75_{10}$**

b) **$101\ 110\ 110\ ,\ 110_2 = 566{,}6_8$**

c) $50{,}25 \times 6 = 301{,}5$ **$301{,}5_{10} = 0110\ \ 0011\ \ 0100,\ 1000_{BCDXS3}$**

10. Konvertovati broj **82,125** iz decimalnog u binarni brojni sistem. Dobijeni binarni broj sabrati sa brojem **100111,011**.

Rešenje:

$\underline{82} : 2 = \underline{41} : 2 = \underline{20} : 2 = \underline{10} : 2 = \underline{5} : 2 = \underline{2} : 2 = \underline{1} : 2 = 0$ **$82_{10} = 1010010_2$**

0 1 0 0 1 0 1

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$0{,}125 \times 2 = 0{,}25$ 0 ↓ **$0{,}125_{10} = 0{,}001$**

$0{,}25 \times 2 = 0{,}5$ 0

$0{,}5 \times 2 = 1{,}0$ 1

```
  100111,011
 1010010,001
 1111001,100
```

11. Brojni primeri za računske operacije sa binarnim brojevima

-- sabiranje:

11001,10	25,5	111,011	7,375
1100,01	12,25	10011,010	19,250
100101,11	37,75	**11010,101**	26,625

-- oduzimanje: a) umanjenik i umanjilac – celi brojevi

20	10100	20	10100
-12	- 1100	-5	- 101
8	**1000**	15	**1111**

b) umanjenik i umanjilac – razlomljeni brojevi

18,25	10010,01	13,125	1101,0010
- 9,50	- 1001,10	- 6,200	- 110,0011
8,75	**1000,11**	6,925	**110,1111**

-- množenje:

101,1 x 10,01
1011
101100
1100,011
5,5 x 2,25 = 12,375

110,01 x 10,1
11001
110010
1111,101
6,25 x 2,5 = 15,625

-- deljenje:

10110,1 : 110,1
101101 : 1101 = **11,011**...
- 1101
10011
- 1101
011000
- 1101
10110
- 1101
1001 ostatak

22,5 : 6,5 = 3,4615...

101,01 : 111,1
1010,1 : 1111 = **0,101...**
10101
- 1111
011000
- 1111
1001 ostatak

5,25 : 7,5 = 0,7

12. Odrediti ukupan broj različitih binarno-kodiranih kodova za BCD kodove:
a) **2421** b) **3321** c} **5211** d) **4221**

Resenje:

a)	2421	b)	3321	c)	5211	d)	4221
0	0000	0	0000	0	0000	0	0000
1	0001	1	0001	1	0001	1	0001
2	0010	2	0010		0010	2	0010
	1000	3	0011	2	0011		0100
3	0011		0100		0100	3	0011
	1001		1000	3	0101		0101
4	0100	4	0101		0110	4	0110
	1010		1001	4	0111		1000
5	1011	5	1010	5	1000	5	1001
	0101		0110	6	1010		0111
6	1100	6	1100		1001	6	1100
	0110		1011	7	1100		1010
7	1101		0111		1011	7	1101
	0111	7	1101	8	1110		1011
8	1110	8	1110		1101	8	1110
9	1111	9	1111	9	1111	9	1111
$2^6 = 64$		**3x2x2x3 = 36**		**$2^6 = 64$**		**$2^6 = 64$**	

II. BULOVA (PREKIDAČKA) ALGEBRA

13. Uprostiti izraz: $\mathbf{Y} = \overline{\overline{A}B} + A\overline{B}C + \overline{A(B+\overline{C})}$

Resenje:

$Y = A + \overline{B} + A\overline{B}C + \overline{A} + BC = A\,(1+\overline{B}\,C) + \overline{A} + \overline{B}\,(1+C)$

$Y = A + \overline{A} + \overline{B} = 1 + \overline{B}$

$\mathbf{Y = 1}$

14. Dokazati identitet: $\overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + A\overline{B}C + AB\overline{C} + ABC = A + \overline{B}C$

Rešenje:

$\overline{A}\,\overline{B}C + A\overline{B}(\overline{C} + C) + AB(\overline{C} + C) = A + \overline{B}C$

$\overline{A}\,\overline{B}C + A(\overline{B} + B) = A + \overline{B}C$

$A + \overline{A}\,\overline{B}\,C = A + \overline{B}\,C$

$(A+\overline{A}\,)\,(A+\overline{B}\,C) = A + \overline{B}\,C$

$\mathbf{A} + \overline{B}\,\mathbf{C = A} + \overline{B}\,\mathbf{C}$

15. Uprostiti izraz**:** **F** = $\overline{X\overline{Y} + XYZ} + \overline{(X + Z)(\overline{X} + Y)}$

Rešenje:

$F = \overline{X\overline{Y}} \cdot \overline{XYZ} + \overline{X + Z} + \overline{\overline{X} + Y} = (\overline{X} + Y)(\overline{X} + \overline{Y} + \overline{Z}) + \overline{X}\,\overline{Z} + X\overline{Y}$

$F = \overline{X} + \overline{X}(\overline{Y} + \overline{Z}) + \overline{X}Y + Y\overline{Y} + Y\overline{Z} + \overline{X}\,\overline{Z} + X\overline{Y} = \overline{X}(1 + \overline{Y} + \overline{Z} + Y + \overline{Z}) + Y\overline{Z} + X\overline{Z} =$

$\overline{X} + Y\overline{Z} + X\overline{Y} = (X + \overline{X}) \cdot (\overline{X} + \overline{Y}) + Y\overline{Z} = \overline{X} + \overline{Y} + Y\overline{Z} = \overline{X}(\overline{Y} + Y) \cdot (\overline{Y} + \overline{Z}) = \overline{X} + \overline{Y} + \overline{Z}$

$F = \overline{XYZ}$

16. Koristeći kombinacionu tabelu, dokazati

a) **A +A·$\overline{\text{B}}$ = A** **b)** $\overline{\textbf{A + BC}}$ **=** $\overline{\textbf{A}}$ **·** $\overline{\textbf{BC}}$

Rešenje:

a)

A	**B**	$\overline{B}$	$A\overline{B}$	$A + A\overline{B}$
0	0	1	0	0
0	1	0	0	0
1	0	1	1	1
1	1	0	0	1

Kolona **A** identična je koloni **A+A**$\overline{B}$ što je i trebalo dokazati.

b)

A	**B**	**C**	**BC**	$\overline{BC}$	$\overline{A}$	**A+BC**	$\overline{A + BC}$	$\overline{A} \cdot \overline{BC}$
0	0	0	0	1	1	0	1	1
0	0	1	0	1	1	0	1	1
0	1	0	0	1	1	0	1	1
0	1	1	1	0	1	1	0	0
1	0	0	0	1	0	1	0	0
1	0	1	0	1	0	1	0	0
1	1	0	0	1	0	1	0	0
1	1	1	1	0	0	1	0	0

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17.1. Za f-ju **Y = (**$\overline{A}$ **+ B)** $\overline{C}$ naći potpunu DF i KF formu.

D.b	A B C	$\overline{A}$	$\overline{A}+B$	$\overline{C}$	**Y**
0.	0 0 0	1	1	1	**1**
1.	0 0 1	1	1	0	**0**
2.	0 1 0	1	1	1	**1**
3.	0 1 1	1	1	0	**0**
4.	1 0 0	0	0	1	**0**
5.	1 0 1	0	0	0	**0**
6.	1 1 0	0	1	1	**1**
7.	1 1 1	0	1	0	**0**

Rešenje:

$\mathbf{Y_{DF}} = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + AB\overline{C}$ $\qquad$ $\mathbf{Y_{DF} = \sum(0, 2, 6)}$

$\mathbf{Y_{KF}} = (A+B+\overline{C})\cdot(A+\overline{B}+\overline{C})\cdot(\overline{A}+B+C)\cdot(\overline{A}+B+\overline{C})\cdot(\overline{A}+\overline{B}+\overline{C})$ $\qquad$ $\mathbf{Y_{KF} = \sum(1, 3, 4, 5. 7)}$

17.2. Za prekidačku f-ju **F = A +** $\overline{BC}$ naći:

a) kombinacionu tabelu
b) kombinacioni vektor
c) potpunu disjunktivnu (DF) i konjuktivnu (KF) formu
d) decimalni skup indeksa za KF i DF

Rešenje:

a)

Dec.br.	**A B C**	**C B**	$\overline{BC}$	**F**
0	0 0 0	0	1	**1**
1	0 0 1	0	1	**1**
2	0 1 0	0	1	**1**
3	0 1 1	1	0	**0**
4	1 0 0	0	1	**1**
5	1 0 1	0	1	**1**
6	1 1 0	0	1	**1**
7	1 1 1	1	0	**1**

b) F = (1,1,1,0,1,1,1,1)

c) $\mathbf{F_{DF}} = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}B\overline{C} + A\overline{B}\,\overline{C} + A\overline{B}C + AB\overline{C} + ABC$

$\mathbf{F_{KF} = A} + \overline{B} + \overline{C}$

d) $\mathbf{F_{DF} = \{0,1,2,4,5,6,7\}}$ $\qquad$ $\mathbf{F_{KF} = \{3\}}$

18. F-je: $\mathbf{Y = BC + A\bar{B}\bar{C}}$ i $\mathbf{Y = \bar{B}(A+\bar{C})}$ predstaviti tabelarno, naći kombinacione vektore, decimalni skup indeksa i potpunu disjunktivnu (DF) i konjuktivnu formu (KF).
(ZADATAK ZA SAMOSTALNO REŠAVANJE)

19. Minimizirati funkciju datu kombinacionom tabelom, primenom aksioma i teorema Bulove algebra.

Rešenje:

d.b	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	1
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

$\mathbf{Y = \sum(1,2,4,5,6)}$

$$Y = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C}$$

$$Y = \bar{B}C(A+\bar{A}) + A\bar{C}(B+\bar{B}) + \bar{A}B\bar{C}$$

$$Y = \bar{B}C + \bar{C}(A+\bar{A}B)$$

$$Y = \bar{B}C + \bar{C}(A+\bar{A})\cdot(A+B)$$

$$\mathbf{Y = \bar{B}C + A\bar{C} + B\bar{C}}$$

$$Y = \bar{B}C + \bar{C}(A+B)$$

20. Minimizirati f-ju iz prethodnog zadatka koristeci Karnoov dijagram.

Rešenje:

A \ BC	00	01	11	10
0	0	1	0	1
1	1	1	0	1

I) $\mathbf{Y = A\bar{C} + \bar{B}C + B}$

II) $\mathbf{Y = A\bar{B} + \bar{B}C + B\bar{C}}$

21. Za f-ju zadatu skupom decimalnih indeksa $\mathbf{Y = \sum(6,7,8,10,11,14,15)}$ naći minimalnu disjunktivnu **(MDF**) i minimalnu konjuktivnu (**MKF**) formu

Rešenje:

	X_1	X_1	$\bar{X}_1$	$\bar{X}_1$	
	0	1	1	0	$\bar{X}_4$
X_2	0	1	1	0	
	0	1	0	0	X_4
$\bar{X}_2$	1	1	0	0	$\bar{X}_4$
	$\bar{X}_3$	X_3	X_3	$\bar{X}_3$	

MDF: $\mathbf{Y = X_2X_3 + X_1X_3 + X_1\bar{X}_2\bar{X}_4}$

MKF: $\mathbf{Y = (\bar{X}_2 + X_3)(X_1 + X_2)(X_3 + \bar{X}_4)}$

Ova varijanta Karnoovog dijagrama nije najpodesnija za učenike pri traženju minimalnih formi, iz dva razloga: prvi – jer je svaki decimalni indeks dat u vidu binarne notacije pa se mora napisati kao proizvod promenljivih i drugi – jer zahteva crtanje novog dijagrama ako se u zadatku traži i konjuktivna forma.

Zato će u ovoj zbirci biti primenjivana druga varijanta, kod koje ne postoje ovi nedostaci.

22. F-ja ima skup decimalnih indeksa **F = ∑(0,1,2,3,6,10,12,13,14)**. Minimizirati f-ju za slučaj da se kombinacije nezavisno promenljivih za decimalne indekse **∑(7,11,15)** nikad ne pojavljuju.

Rešenje:

CD \ AB	00	01	11	10
00	1	0	1	0
01	1	0	1	0
11	1	-	-	-
10	1	1	1	1

$$\mathbf{Y = C + \overline{A}\,\overline{B} + AB}$$

23. Za f-ju zadatu skupom decimalnih indeksa **F(A,B,C,D) = {(6,7,8,9,10,11,15)** naći **MDF**.

Rešenje:

Prvo rešenje: $\mathbf{F = A\overline{B} + BCD + \overline{A}BC}$

Drugo rešenje: $\mathbf{F = A\overline{B} + \overline{A}BC + ACD}$

24. F-ja **F(A,B,C,D)** zadata je Karnoovim dijagramom:

a) minimizirati datu f-ju
b) naći skup decimalnih indeksa za koje f-ja ima vrednost 1 i skup indeksa za koje f-ja nije definisana

Rešenje:

CD \ AB	00	01	11	10
00	0	1	1	0
01	0	-	-	1
11	-	0	-	1
10	0	1	1	0

a) $\mathbf{Y = B\overline{D} + AD}$

b) $\mathbf{Y_1 = \sum(4,6,9,11,12,14)}$

$\mathbf{Y_- = \sum(3,5,13,15)}$

25. Koristeći Karnoov dijagram naći **MDF** za f-ju: **F(A,B,C,D) =** **$\sum$ (1,4,5,6,8,12,13,15).**

Rešenje:

CD \ AB	00	01	11	10
00	0	1	1	1
01	1	1	1	0
11	0	0	1	0
10	0	1	0	0

$$F = \overline{A}\,\overline{C}\,D + ABD + A\overline{C}\,\overline{D} + \overline{A}\,B\,\overline{D}$$

26. Prekidačka f-ja data je Karnoovim dijagramom. Naći minimalne disjunktivne forme.

a)

CD \ AB	00	01	11	10
00	1	0	1	1
01	1	0	1	1
11	1	1	1	0
10	1	1	1	0

Rešenja:

I) $F = AB + \overline{B}\,\overline{C} + \overline{A}\,C$

II) $F = BC + A\overline{C} + \overline{A}\,\overline{B}$

b)

CD \ AB	00	01	11	10
00	-	0	0	1
01	1	0	0	1
11	1	1	1	1
10	1	0	-	0

$$F = \overline{B}\,\overline{C} + CD + \overline{A}\,\overline{B}$$

27. Sintetizovati (projektovati) automat za nezavisno paljenje i gašenje sijalica sa ma koga od tri različita mesta, koristeci relejna logicka kola.

Rešenje:

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

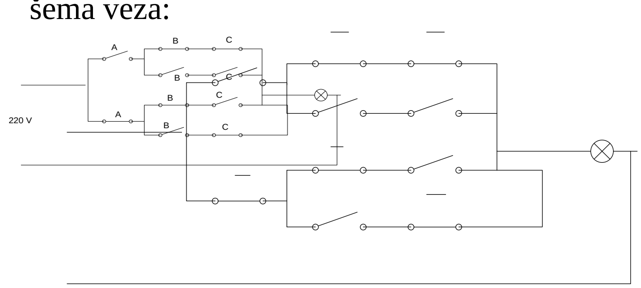
minimizacija:

C \ AB	00	01	11	10
0	0	1	0	1
1	1	0	1	0

$$Y = \overline{A}\,\overline{B}C + \overline{A}B\overline{C} + A\overline{B}\,\overline{C} + ABC$$

$$Y = \overline{A}(\overline{B}C + B\overline{C}) + A(\overline{B}\,\overline{C} + AB)$$

šema veza:



28. Realizovati šemu prekidačke mreze preko koje može da se pali sijalica sa tri različita mesta nezavisno, koristeći relejnu tehniku.

Rešenje:

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

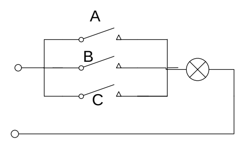
	AB			
	00	01	11	10
C 0	0	1	1	1
1	1	1	1	1

$$\mathbf{Y = A + B + C}$$

šema mreže:

-bez Karnoovog dijagrama:

$$\mathbf{Y_{KF} = A + B + C}$$



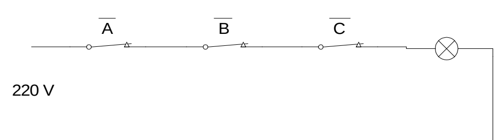
29. Realizovati automat za gašenje sijalice sa tri različita mesta nezavisno, koristeći relejnu tehniku.

Rešenje:

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

$$\mathbf{Y_{DF}} = \overline{A}\,\overline{B}\,\overline{C}$$

Šema:



30. Kod **BCD koda “više 3” (BCDXS3)** sa kontrolom parnosti smatrati da su cifre kodnog sloga označene sa A,B,C i D, a cifra parnosti sa Y. Naci **Y = F(A,B,C,D).** Napisati u vidu obe normalne forme.

Rešenje:

db	A	B	C	D	Y
0	0	0	1	1	0
1	0	1	0	0	1
2	0	1	0	1	0
3	0	1	1	0	0
4	0	1	1	1	1
5	1	0	0	0	1
6	1	0	0	1	0
7	1	0	1	0	0
8	1	0	1	1	1
9	1	1	0	0	0

NDF: Y = {(1,4,5,8)

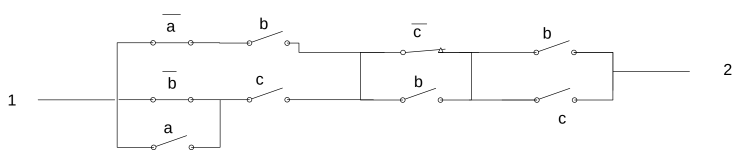
$$Y = \overline{A}B\overline{C}\,\overline{D} + \overline{A}BCD + A\overline{B}\,\overline{C}\,\overline{D} + A\overline{B}CD$$

NKF: Y = {(0,2,3,6,7,9)

$$Y = (A + B + \overline{C} + \overline{D}) \cdot (A + \overline{B} + C + \overline{D}) \cdot (A + \overline{B} + \overline{C} + D)$$
$$\cdot (\overline{A} + B + C + \overline{D}) \cdot (\overline{A} + B + \overline{C} + D) \cdot (\overline{A} + \overline{B} + C + D)$$

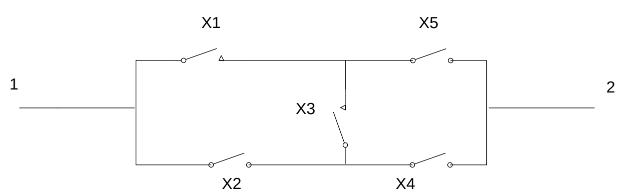
31. Analizirati bilateralnu mrežu, t.j naći Y_{12}.

a)



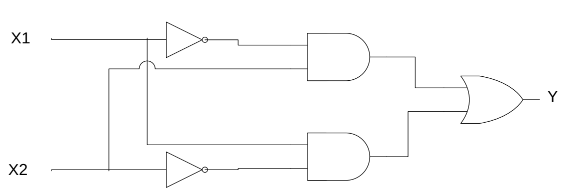
$$\mathbf{Y_{12} = [(a + \overline{b})\, c + \overline{a}\, b\,]\,(b + \overline{c})\,(b + c)}$$

b)



$$\mathbf{Y_{12} = X_1X_5 + X_2X_4 + X_1X_3X_4 + X_2X_3X_5}$$

32. Analizirati unilateralnu mrežu:



Rešenje: $\mathbf{Y = \overline{X}_1X_2 + X_1\overline{X}_2}$

33. Za f-ju zadatu skupom decimalnih indeksa **$F = \sum(2,3,7,10,14,15)$, $n = 4$,** konstruisati minimalne bilateralne mreže.

Rešenje:

CD \ AB	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	1	1	1	0
10	1	0	1	1

a) $\mathbf{Y_{12}} = \bar{A}\bar{B}C + BCD + AC\bar{D}$

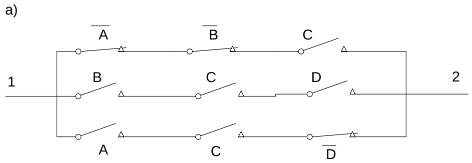
$\mathbf{Y_{12}} = C(\bar{A}\bar{B} + BD + A\bar{D})$

b) $\mathbf{Y_{12}} = \bar{A}CD + ABC + \bar{B}C\bar{D}$

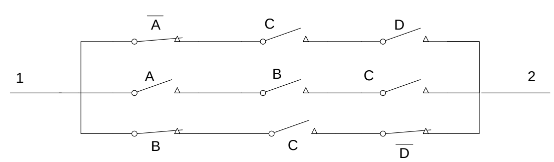
$\mathbf{Y_{12}} = C(\bar{A}D + AB + \bar{B}\bar{D})$

c) $\mathbf{Y_{KF}} = C \cdot (A + \bar{B} + C) \cdot (\bar{A} + B + \bar{D})$

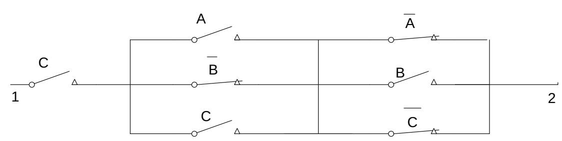
a)



b)



c)



34. F-ja **$Y = f(A.B.C.D)$** ima skup decimalnih indeksa **$\sum(0,1,4,6,10,11,12,14,15)$.**

a) minimizirati f-ju
b) minimizirati f-ju za slučaj da članovi normalne forme koji odgovaraju indeksima iz skupa **$\sum(5,7,13)$** mogu proizvoljno imati vrednost 0 ili 1
c) realizovati f-je pod **a) i b)** pomocu logickih kola **I, ILI, NE**

Rešenje:

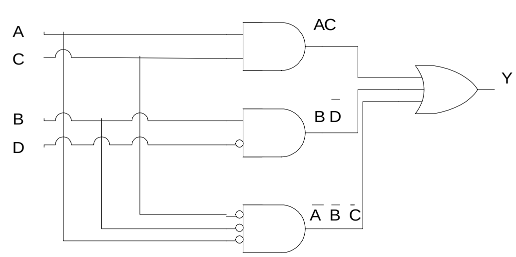
a)

CD \ AB	00	01	11	10
00	1	1	1	0
01	1	0	0	0
11	0	0	1	1
10	0	1	1	1

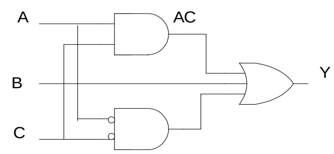
b)

CD \ AB	00	01	11	10
00	1	1	1	0
01	1	-	-	0
11	0	-	1	1
10	0	1	1	1

a) **$Y = AC + \bar{A}\bar{B}\bar{C} + B\bar{D}$**



b) $\mathbf{Y = B + AC} + \overline{A}\,\overline{C}$



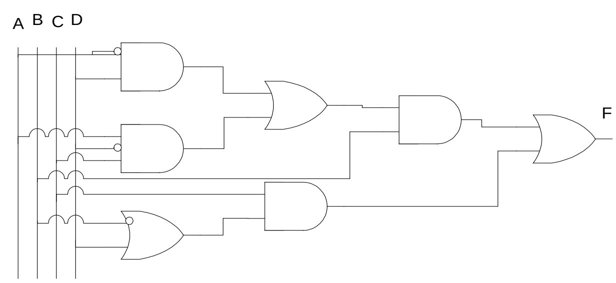
35. Naci minimalnu **DF** f-je **F(A,B,C,D) = ∑(0,1,5,7,8,9,13,14)** i realizovati logičko-prekidačku mrežu sa elementarnim logičkim kolima.

Rešenje:

CD \ AB	00	01	11	10
00	1	0	0	1
01	1	1	1	1
11	0	1	0	0
10	0	0	1	0

$F = \overline{C}D + \overline{B}\,\overline{C} + \overline{A}BD + ABC\overline{D}$

$\mathbf{F} = \overline{C}(D + \overline{B}) + B(\overline{A}D + AC\overline{D})$



36. Data je f-ja **F(A,B,C,D) = ∑(0,1,3,7,8,9,10,11,14,15).** Koristeći Karnoov dijagram odrediti:

a) **MDF** b) **MKF** c) realizovati prekidačku mrežu sa **I, ILI i NE** kolima

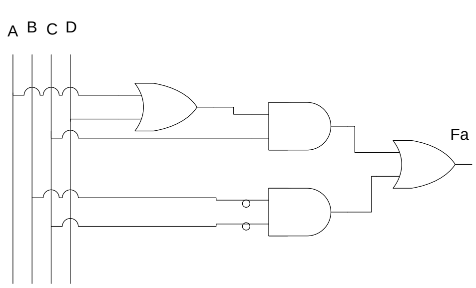
Rešenje:

CD \ AB	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	1	1	1
10	0	0	1	1

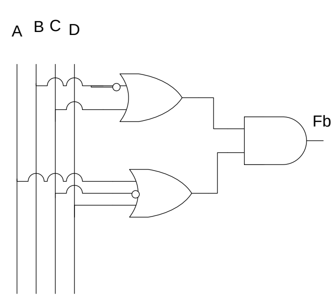
a) $\mathbf{F} = \overline{B}\,\overline{C} + \mathbf{CD + AC} = \overline{B}\,\overline{C} + \mathbf{C(A+D)}$

b) $\mathbf{F} = (\overline{B} + \mathbf{C})\,(\mathbf{A} + \overline{C} + \mathbf{D})$

c)



C.a)



C.b)

37. Data je f-ja kombinacionom tabelom

a) napisati potpunu DF   b) napisati potpunu KF

c) realizovati logičku mrežu koja realizuje date f-je u minimalnoj formi

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

Rešenje:

a) $\mathbf{F} = \bar{A}\mathbf{B}\bar{C} + \bar{A}\mathbf{BC} + \mathbf{AB}\bar{C}$

b) $\mathbf{F} = (A+B+C)(A+B+\bar{C})(\bar{A}+B+C)(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+\bar{C})$

c)

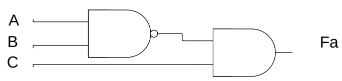
A \ BC	00	01	11	10
0	0	1	1	0
1	0	1	0	0

$c_{a)}$ $\mathbf{F} = \bar{A}\mathbf{C} + \bar{B}\mathbf{C}$

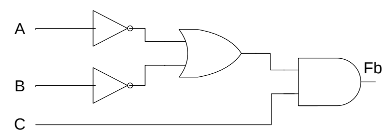
$\mathbf{F} = \mathbf{C}\cdot(\bar{A}+\bar{B}) =$

$= \mathbf{C}\cdot\overline{AB}$

$c_{b)}$ $\mathbf{F} = \mathbf{C}(\bar{A}+\bar{B}) = \mathbf{C}\cdot\overline{AB}$



c.a)



c.b)

38. Realizovati prekidačku mrežu koja ostvaruje f-ju **F(A,B,C),** čija je vrednost kontrola parnosti kodnog sloga.

Rešenje:

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

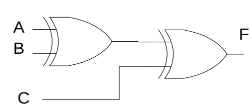
C \ AB	00	01	11	10
0	0	1	0	1
1	1	0	1	0

$F = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$

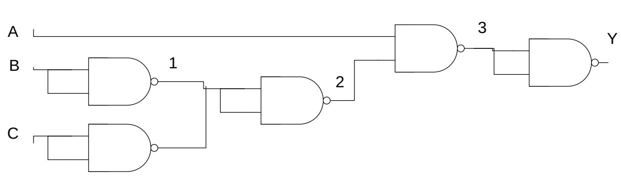
$F = \bar{A}(\bar{B}C + B\bar{C}) + A(\bar{B}\bar{C} + BC)$

$F = \bar{A}(B\oplus C) + A(\overline{B\oplus C})$

$\mathbf{F = A \oplus (B \oplus C)}$



39. Analiziraj logičko-prekidačku mrežu:



Rešenje:

$Y_1 = \overline{B}\,\overline{C} = \overline{B+C}$ $\quad Y_2 = B+C \quad$ $Y_3 = \overline{A\cdot(B+C)}$ $\quad$ **$Y = \overline{Y}_3 = A(B+C)$**

40. Realizovati f-je zadate Karnoovim dijagramima:

a)

BA \ DC	00	01	11	10
00	1	1	0	*
01	0	0	1	*
11	1	*	0	0
10	0	*	1	1

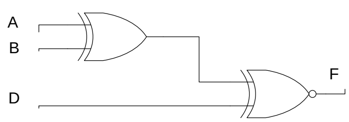
Rešenje:

$F = \overline{A}\,\overline{B}\,\overline{D} + AB\overline{D} + \overline{A}BD + A\overline{B}D$

$F = \overline{D}(\overline{A}\,\overline{B} + AB) + D(\overline{A}B + A\overline{B})$

$F = \overline{D}\,\overline{(A\oplus B)} + D(A\oplus B)$ $\qquad$ A+B = Y

$F = \overline{D}\,\overline{Y} + D\,Y = \overline{D\oplus Y}$ $\qquad$ $F = \overline{D\oplus(A\oplus B)}$

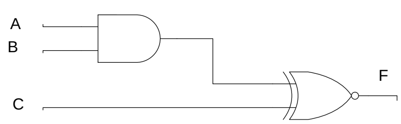


b)

BA \ DC	00	11	11	10
00	*	0	0	1
01	1	0	*	1
11	0	1	*	0
10	*	0	*	1

$F = \overline{B}\,\overline{C} + \overline{A}\,\overline{C} + ABC = \overline{C}(\overline{A}+\overline{B}) + C\cdot AB$

Y = AB $\qquad$ $\overline{AB} = \overline{A}+\overline{B} = \overline{Y}$ $\qquad$ $F = \overline{C}\,\overline{Y} + CY = \overline{C\oplus Y} = \overline{C\oplus(AB)}$

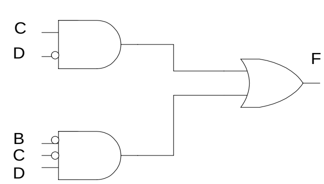


41. Zadata je f-ja: **F(A,B,C,D) = ∑(1,2,6,9,10,14) + ∗ (7,8,12).** Napisati minimalne forme – **MDF i MKF** i realizovati f-je. <u>Koja je forma povoljnija za realizaciju?</u>

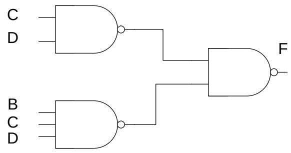
Rešenje:

CD \ AB	00	01	11	10
00	0	0	*	*
01	1	0	0	1
11	0	*	0	0
10	1	1	1	1

$\mathbf{F_{DF} = C\overline{D} + \overline{B}\,\overline{C}D}$



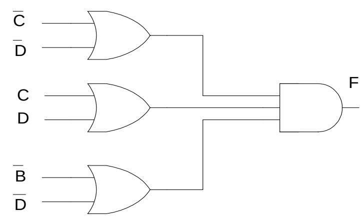
sa NI kolima: **F_{DF} =** $\overline{\overline{C\overline{D} + \overline{B}\,\overline{C}D}} = \overline{\overline{C\overline{D}} \cdot \overline{\overline{B}\,\overline{C}D}}$



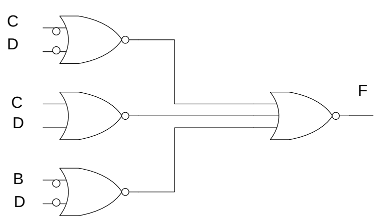
AB

CD \ AB	00	01	11	10
00	0	0	*	*
01	1	0	0	1
11	0	*	0	0
10	1	1	1	1

F_{KF} = ($\overline{C} + \overline{D}$ **) (C+D) (** $\overline{B} + \overline{D}$ **)**

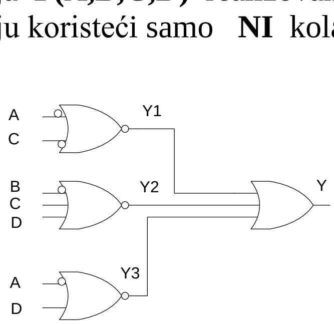


sa NILI kolima: $F_{KF} = \overline{\overline{(\overline{C}+\overline{D})\cdot(C+D)\cdot(\overline{B}+\overline{D})}}$ $F_{KF} = \overline{\overline{\overline{C}+\overline{D}} + \overline{C+D} + \overline{\overline{B}+\overline{D}}}$



U ovom primeru povoljnije je resenje MDF, jer se realizuje sa manje logičkih elemenata.

42. F-ja **F(A,B,C,D)** realizovana je sa **NILI-ILI** kolima prema slici. Realizovati datu f-ju koristeći samo **NI** kola.

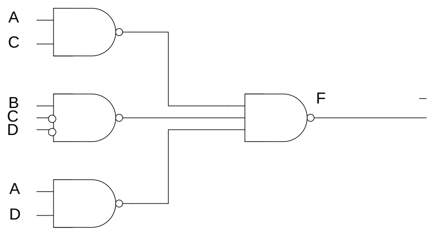


Rešenje:

$$Y_1 = \overline{\overline{A}+\overline{C}} \qquad Y_2 = \overline{\overline{B}+C+D} \qquad Y_3 = \overline{\overline{A}+\overline{D}}$$

$$F = Y_1 + Y_2 + Y_3 = \overline{\overline{A}+\overline{C}} + \overline{\overline{B}+C+D} + \overline{\overline{A}+\overline{D}}$$

$$F = \overline{\overline{AC + B\overline{C}\overline{D} + AD}} = \overline{\overline{AC}\cdot\overline{B\overline{C}\overline{D}}\cdot\overline{AD}}$$



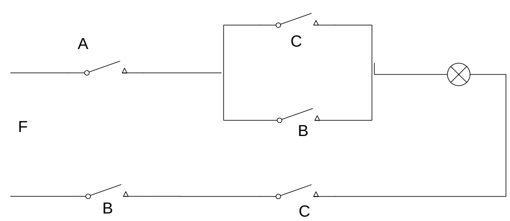
43. Komisija od tri člana glasa o izboru kandidata pritiskom na taster u slučaju pozitivnog glasa (ako je za). Prima se kandidat sa većinom glasova. Formirati prekidačku mrežu koja pali kontrolnu lampu u slučaju izbora kandidata.

Rešenje:

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

F = AC + BC + AB = **A(C+B) + BC**

	BC 00	01	11	10
A 0	0	0	1	0
1	0	1	1	1



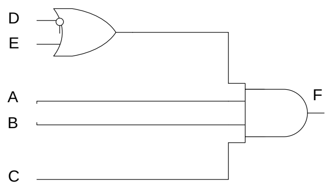
44. Projektovati logiku mreže koja će da omogući paljenje motora automobila pod sledećim uslovima: da je ključ za paljenje motora u bravi (A) ; da je vozač seo na svoje sediste (B); i da je prikopčao sigurnosni pojas (C); da je sediste suvozača prazno (D) ili da suvozač sedi na svom sedištu sa prikopčanim pojasom (E).

Rešenje:

F(A,B,C,D,E) = A B C ($\overline{D}$ + DE) =

=A B C ($\overline{D}$ +D) ($\overline{D}$ +E)) =

=**A B C (** $\overline{D}$ **+E)**



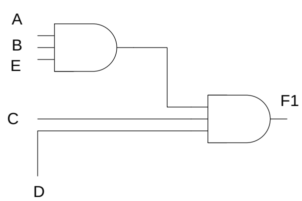
45. Odrediti logiku mreže koja će upravljati uredjajem za otvaranje brave (napr. na kasi) pomoću kartice sa **5** otvora tako, da se brava otvora na šifru **11001** i da se aktivira alarm pri ubacivanju bilo koje druge kartice.

Rešenje:

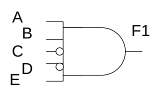
F-ja za otvaranje brave je: $\mathbf{F_1 = A\,B\,\bar{C}\,\bar{D}\,E}$

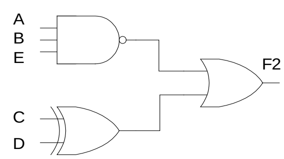
F-ja uključivanje alarma je: $F_2 = \overline{AB\bar{C}\bar{D}E + ABCDE} = \overline{ABE(\bar{C}\bar{D} + CD)} =$

$$= \overline{ABE} + \overline{\bar{C}\bar{D} + CD} = \overline{ABE} + C \oplus D$$



ILI





46. F-je: a) $\mathbf{F(a,b,c,d) = \sum(0,1,4,6,10,12,14,15) + {}^{*}(5,7,11,13)}$

b) $\mathbf{F(A,B,C,D) = \sum(0,1,2,5,7,9) + {}^{*}(8,10,14)}$

realizovati pomoću univerzalnih logičkih kola.

ZADATAK ZA SAMOSTALNO REŠAVANJE

III. KOMBINACIONE MREŽE

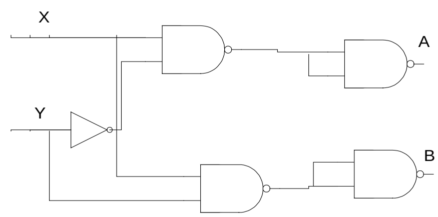
47. Sastaviti kodersku matricu za kod **2421**

48. Sastaviti dekodersku matricu za kod **2421**

49. Sastaviti distributor **DMX 1/2** koristeći NI kola sa dva ulaza

Rešenje:

Y	X	A	B
0	0	0	0
0	1	1	0
1	0	0	0
1	1	0	1



50. Projektovati demultiplekser **DMX 1/8 .**

Rešenje:

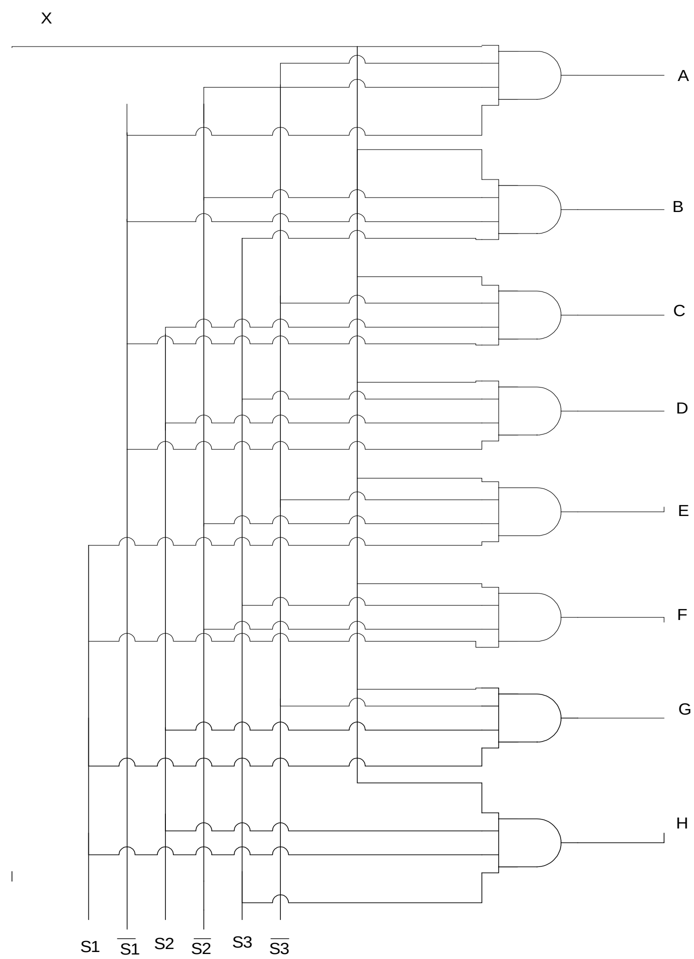
S_1	S_2	S_3	A	B	C	D	E	F	G	H
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

$A = \overline{S}_1 \overline{S}_2 \overline{S}_3 \ X$ $B = \overline{S}_1 \overline{S}_2 S_3 \ X$

$C = \overline{S}_1 S_2 \overline{S}_3 \ X$ $D = \overline{S}_1 S_2 S_3 \ X$

$E = S_1 \overline{S}_2 \overline{S}_3 \ X$ $F = S_1 \overline{S}_2 S_3 \ X$

$G = S_1 S_2 \overline{S}_3 \ X$ $H = S_1 S_2 S_3 \ X$



51. Realizovati adresni dekoder od demultipleksera **1/8**

Rešenje:

Ulazi			Izlazi							
S_1	S_2	X	I_0	I_1	I_2	I_3	I_4	I_5	I_6	I_7
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

$I_0 = \bar{S}_1 \bar{S}_2 \bar{X}$

$I_1 = \bar{S}_1 \bar{S}_2 X$

$I_2 = \bar{S}_1 S_2 \bar{X}$

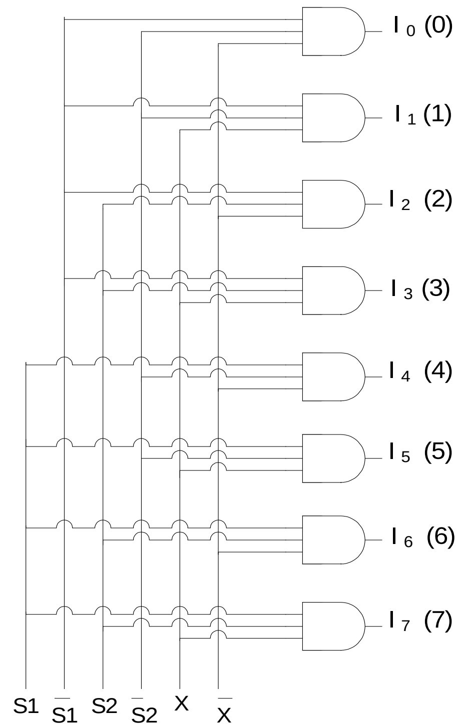
$I_3 = \bar{S}_1 S_2 X$

$I_4 = S_1 \bar{S}_2 \bar{X}$

$I_5 = S_1 \bar{S}_2 X$

$I_6 = S_1 S_2 \bar{X}$

$I_7 = S_1 S_2 X$



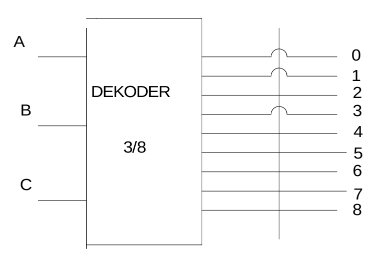
52. Realizovati f-ju **F(A,B,C) = A + B**$\bar{C}$ pomoću dekodera **3/8.**

Rešenje:

A	B	C	F	izl.
0	0	0	0	0
0	0	1	0	1
0	1	0	1	2
0	1	1	0	3
1	0	0	1	4
1	0	1	1	5
1	1	0	1	6
1	1	1	1	7

Potpuna disjunktivna forma f-je je:

$F = \bar{A}B\bar{C} + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC = \sum(2\ 4,5,6,7)$



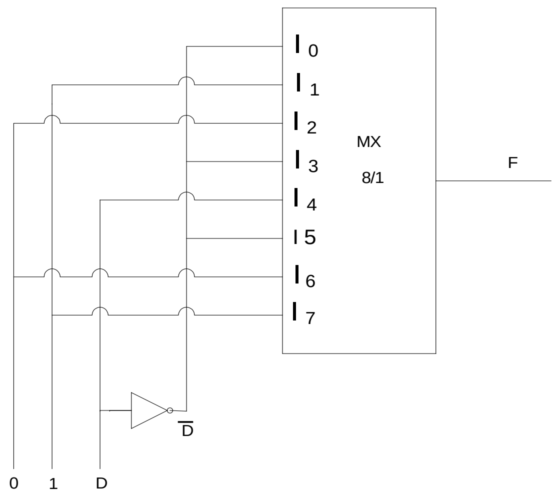
53. F-ju **F(A,B,C,D) = ∑(0,2,3,6,9,10,14,15)** realizovati pomoću multipleksera **MX 8/1**
Za selekcione ulaze uzeti promenljive **A,B** i **C.**

Rešenje:

$$F = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}CD + \bar{A}BC\bar{D} + A\bar{B}\bar{C}D + A\bar{B}C\bar{D} + ABC\bar{D} + ABCD =$$

$$= \bar{A}\bar{B}\bar{C}(\bar{D}) + \bar{A}\bar{B}C(\bar{D} + D) + \bar{A}BC(\bar{D}) + A\bar{B}\bar{C}(D) + A\bar{B}C(\bar{D}) + ABC(\bar{D} + D)$$

A B C	ostatak	
0 0 0	$\bar{D}$	I_0
0 0 1	1	I_1
0 1 0	0	I_2
0 1 1	$\bar{D}$	I_3
1 0 0	D	I_4
1 0 1	$\bar{D}$	I_5
1 1 0	0	I_6
1 1 1	1	I_7



54. F-ju iz zad.**53.** realizovati pomoću multipleksera **MX 4/1 .** Za selekcione ulaze uzeti promenljive **A** i **B.**

Rešenje:

$$F = \bar{A}\bar{B}(\bar{C}\bar{D} + C\bar{D} + CD) + \bar{A}B(C\bar{D}) + A\bar{B}(\bar{C}D + C\bar{D}) + AB(C\bar{D} + CD)$$

I_0 I_1 I_2 I_3

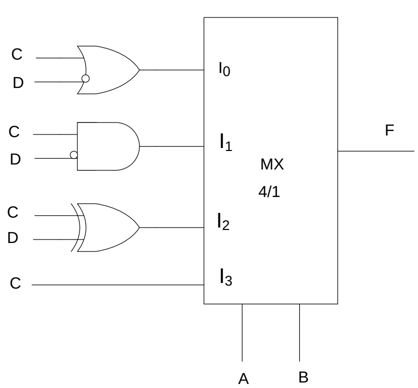
A B	ostatak	I
0 0	$C+\bar{D}$	I_0
0 1	$C\bar{D}$	I_1
1 0	$C \oplus D$	I_2
1 1	C	I_3

$I_0 = \bar{C}\bar{D} + C(\bar{D} + D) = C + \bar{C}\bar{D} = (C + \bar{C}) \cdot (C + \bar{D}) = C + \bar{D}$

$I_1 = C\bar{D}$

$I_2 = \bar{C}D + C\bar{D} = C \oplus D$

$I_3 = C(\bar{D} + D) = C$



55. Realizovati f-ju **F(A,B,C,D) = ∑(0,3,4,5,9,10,12,13)** pomoću multipleksera **MX 8/1** u slucaju da su adresne promenljive **A,C i D** , ili pomocu **MX 4/1** i logickih kola. ako su selekcioni ulazi **A i B.**

Rešenje:

a)

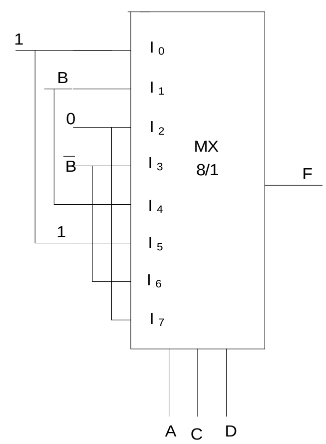
$0 - \bar{A}\bar{B}\bar{C}\bar{D}$, $3 - \bar{A}\bar{B}CD$, $4 - \bar{A}B\bar{C}\bar{D}$, $5 - \bar{A}B\bar{C}D$, $9 - A\bar{B}\bar{C}D$, $10 - A\bar{B}C\bar{D}$,

$12 - AB\bar{C}\bar{D}$, $13 - AB\bar{C}D$

$$F = \bar{A}\bar{C}\bar{D}\cdot(\bar{B}+B) + \bar{A}CD\cdot(\bar{B}) + \bar{A}\bar{C}D\cdot(B) + A\bar{C}D\cdot(\bar{B}+B) + A\bar{C}\bar{D}\cdot(B) + AC\bar{D}\cdot(\bar{B})$$

$$F = \bar{A}\bar{C}\bar{D}\ (1) + \bar{A}CD\ (\bar{B}) + \bar{A}\bar{C}D\ (B) + A\bar{C}D\ (1) + A\bar{C}\bar{D}\ (B) + AC\bar{D}\ (B)$$

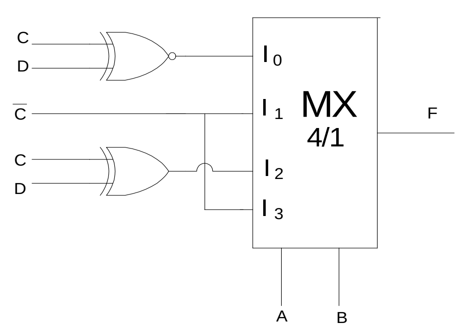
A	C	D	ostatak	I
0	0	0	1	I_0
0	0	1	B	I_1
0	1	0	0	I_2
0	1	1	$\bar{B}$	I_3
1	0	0	B	I_4
1	0	1	1	I_5
1	1	0	$\bar{B}$	I_6
1	1	1	0	I_7



b) $F = \bar{A}\bar{B}\cdot(\bar{C}\bar{D} + CD) + \bar{A}B\cdot(\bar{C}\bar{D} + \bar{C}D) + A\bar{B}\cdot(\bar{C}D + C\bar{D}) + AB\cdot(\bar{C}\bar{D} + \bar{C}D)$

$$F = \bar{A}\bar{B}\cdot(\overline{C \oplus D}) + \bar{A}B\cdot\bar{C} + A\bar{B}\cdot(C \oplus D) + AB\cdot\bar{C}$$

A	B	ostatak	I
0	0	$\overline{C \oplus D}$	I_0
0	1	$\bar{C}$	I_1
1	0	$C \oplus D$	I_2
1	1	$\bar{C}$	I_3



56. Funkciju **F(A,B,C) = A + B**$\overline{C}$ realizovati pomoću multipleksera **MX 2/1** i logičkih kola. Za selekcioni ulaz uzeti promenljivu **A.**

Rešenje:

Db	A	B	C	C	BC	F
0	0	0	0	1	0	0
1	0	0	1	0	0	0
2	0	1	0	1	1	1
3	0	1	1	0	0	0
4	1	0	0	1	0	1
5	1	0	1	0	0	1
6	1	1	0	1	1	1
7	1	1	1	0	0	1

$$F = \sum (2,4,5,6,7)$$

$$F = \overline{A}B\overline{C} + A\overline{B}\overline{C} + A\overline{B}C + AB\overline{C} + ABC$$

$$F = \overline{A}\cdot(B\overline{C}) + A\cdot(\overline{B}\overline{C} + \overline{B}C + B\overline{C} + BC)$$

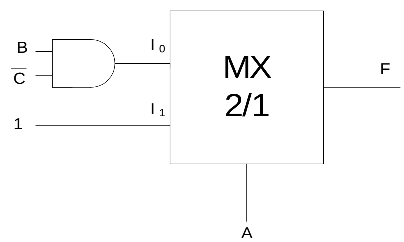
$$F = \overline{A}\cdot(B\overline{C}) + A\cdot(\overline{B}\overline{C} + BC + \overline{B}C + B\overline{C})$$

$$F = \overline{A}\cdot(B\overline{C}) + A\cdot(\overline{B\oplus C} + B\oplus C) = \overline{A}\cdot(B\overline{C}) + A\cdot(1)$$

1

Tabela

A	ost.	I
0	$B\overline{C}$	I_0
1	1	I_1



57. Realizovati konvertor koda **2421** (Ajkenov) u **BCD 8421** kod, koristeći:
a) elementarna logička kola b) univerzalna NI kola

Resenje:

dec. br.	2 4 2 1 DCBA	NBCD ZYXW
0	0 0 0 0	0 0 0 0
1	0 0 0 1	0 0 0 1
2	0 0 1 0	0 0 1 0
3	0 0 1 1	0 0 1 1
4	0 1 0 0	0 1 0 0
5	1 0 1 1	0 1 0 1
6	1 1 0 0	0 1 1 0
7	1 1 0 1	0 1 1 1
8	1 1 1 0	1 0 0 0
9	1 1 1 1	1 0 0 1

a)

W = A

X (DC / BA)

BA \ DC	00	01	11	10
00	0	0	1	-
01	0	-	1	-
11	1	-	0	0
10	1	-	0	-

Y (DC / BA)

BA \ DC	00	01	11	10
00	0	1	1	-
01	0	-	1	-
11	0	-	0	1
10	0	-	0	-

X = $B\overline{D} + \overline{B}D = \mathbf{B\oplus D}$

Y = $\overline{B}\mathbf{C} + \mathbf{D}\overline{C}$

Z (DC / BA)

BA \ DC	00	01	11	10
00	0	0	0	-
01	0	-	0	-
11	0	-	1	0
10	0	-	1	-

Z = BC

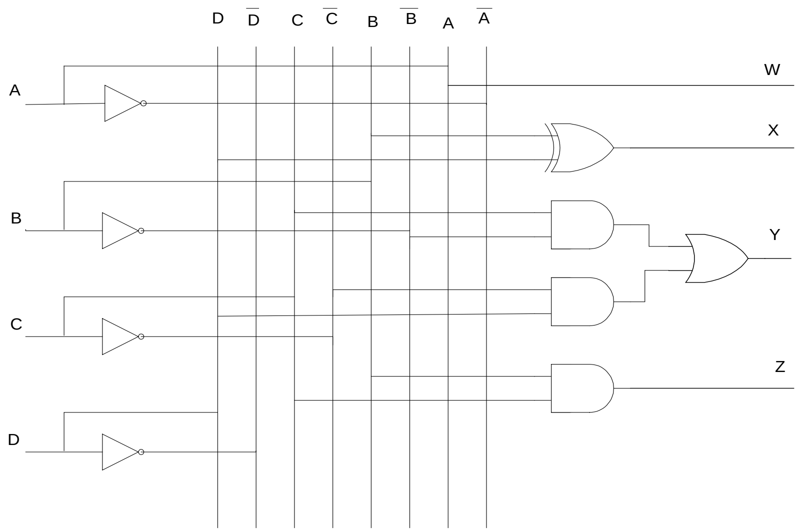
b) $\overline{\overline{X}} = \overline{\overline{B\overline{D} + \overline{B}D}} = \overline{\overline{B\overline{D}}\cdot\overline{\overline{B}D}}$ = **X**

W =A

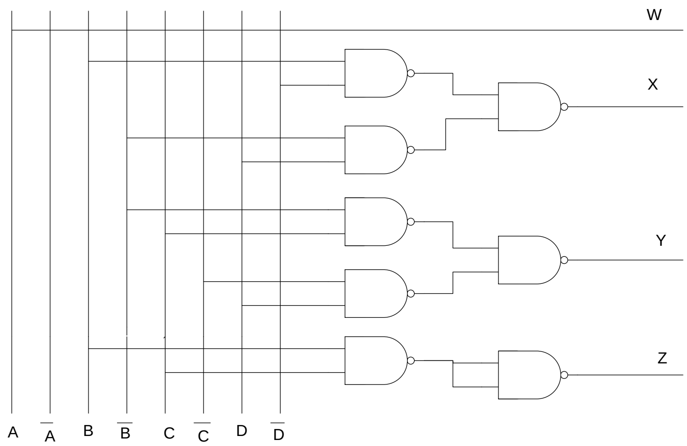
Y = $\overline{\overline{B}C + D\overline{C}} = \overline{\overline{\overline{B}C}\cdot\overline{D\overline{C}}}$

Z = $\overline{\overline{BC}}$

a)



b)



58. Realizovati konvertor koda **NBCD** u kod **4221** , koristeći elementarna logička kola.

Rešenje:

dec. br.	NBCD kod D C B A	kod 4221 Z Y X W
0	0 0 0 0	0 0 0 0
1	0 0 0 1	0 0 0 1
2	0 0 1 0	0 0 1 0
3	0 0 1 1	0 0 1 1
4	0 1 0 0	0 1 1 0
5	0 1 0 1	0 1 1 1
6	0 1 1 0	1 1 0 0
7	0 1 1 1	1 1 0 1
8	1 0 0 0	1 1 1 0
9	1 0 0 1	1 1 1 1

W = A

X:

BA \ DC	00	01	11	10
00	0	1	-	1
01	0	1	-	1
11	1	0	-	-
10	1	0	-	-

Y:

BA \ DC	00	01	11	10
00	0	1	-	1
01	0	1	-	1
11	0	1	-	-
10	0	1	-	-

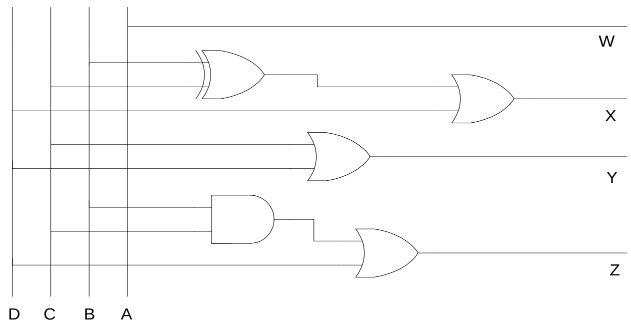
X $= D + B\overline{C} + \overline{B}C =$
$= D + B \oplus C$

Y = C + D

Z:

BA \ DC	00	01	11	10
00	0	0	-	1
01	0	0	-	1
11	0	1	-	-
10	0	1	-	-

Z = D + BC



59. Realizovati konvertor a) **Grejovog koda** u kod **BCD 8421** b) koda **BCD 8421** u **Grejov kod.**

Rešenje:

a)

db.	Grejov kod D	C	B	A	BCD 8421 Z	Y	X	W
0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	1
2	0	0	1	1	0	0	1	0
3	0	0	1	0	0	0	1	1
4	0	1	1	0	0	1	0	0
5	0	1	1	1	0	1	0	1
6	0	1	0	1	0	1	1	0
7	0	1	0	0	0	1	1	1
8	1	1	0	0	1	0	0	0
9	1	1	0	1	1	0	0	1
10	1	1	1	1	-	-	-	-
11	1	1	1	0	-	-	-	-
12	1	0	1	0	-	-	-	-
13	1	0	1	1	-	-	-	-
14	1	0	0	1	-	-	-	-
15	1	0	0	0	-	-	-	-

Z = D

Y:

BA \ DC	00	01	11	10
00	0	1	0	-
01	0	1	0	-
11	0	1	-	-
10	0	1	-	-

X:

BA \ DC	00	01	11	10
00	0	1	0	-
01	0	1	0	-
11	1	0	-	-
10	1	0	-	-

Y = $\overline{D}C$

X $= \overline{C}B + \overline{D}C\overline{B} = \overline{C}B + Y\overline{B}$

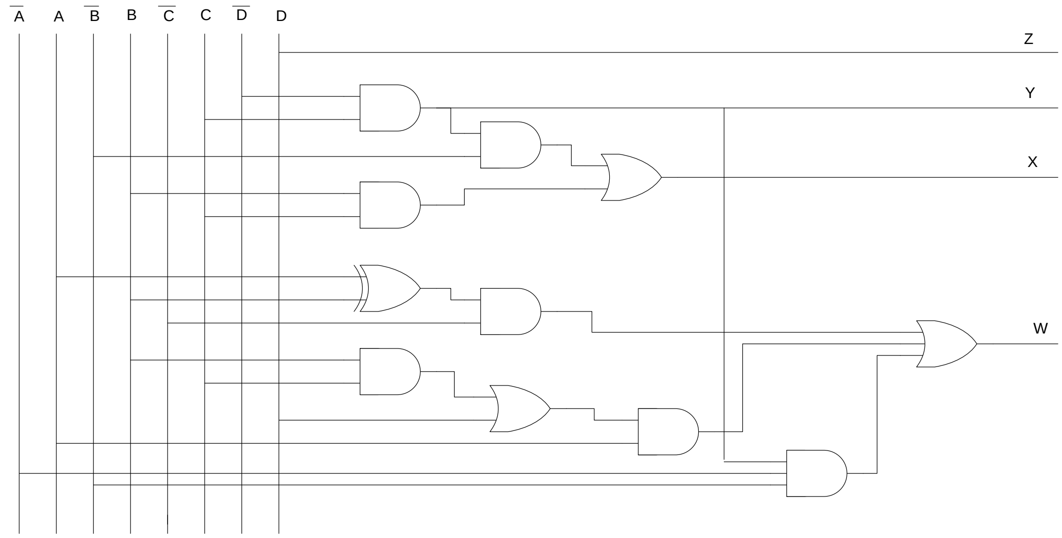
W:

BA \ DC	00	01	11	10
00	0	1	0	-
01	1	0	1	-
11	0	1	-	-
10	1	0	-	-

W $= \overline{C}\overline{B}A + \overline{C}B\overline{A} + DA + CBA +$
$+ \overline{D}C\overline{B}\overline{A}$

$W = \overline{C}(\overline{B}A + B\overline{A}) + A(D+CB) +$
$+ \overline{D}C\overline{B}\overline{A}$

W = $\mathbf{\overline{C}(A \oplus B) + A(D+CB) + Y\overline{B}\overline{A}}$



b)

A: (rows: xw, columns: zy)

xw \ zy	00	01	11	10
00	0	0	-	0
01	1	1	-	1
11	0	0	-	-
10	1	1	-	-

B: (rows: xw, columns: zy)

xw \ zy	00	01	11	10
00	0	1	-	0
01	0	1	-	0
11	1	0	-	-
10	1	0	-	-

C:

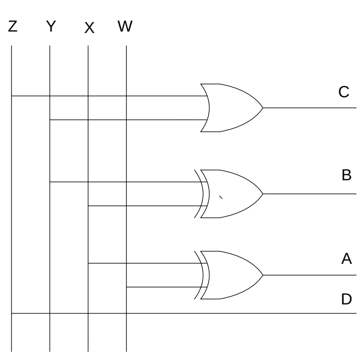
xw \ ZY	00	01	11	10
00	0	1	-	1
01	0	1	-	1
11	0	1	-	-
10	0	1	-	-

D = Z **C = Y+Z**

$A = \overline{X}W + X\overline{W}$ **A = X ⊕ W**

$B = \overline{Y}X + Y\overline{X}$

B = X ⊕ Y



60. Realizovati konvertor a) binarnog u Grejov kod b) Grejovog u binarni kod (uzeti trobitne kodove).

Rešenje:

db.	B_2	B_1	B_0	G_2	G_1	G_0
0	0	0	0	0	0	0
1	0	0	1	0	0	1
2	0	1	0	0	1	1
3	0	1	1	0	1	0
4	1	0	0	1	1	0
5	1	0	1	1	1	1
6	1	1	0	1	0	1
7	1	1	1	1	0	0

a)

$$\mathbf{G_2 = B_2}$$
$$\mathbf{G_1 = B_1 \oplus B_2}$$
$$\mathbf{G_0 = B_0 \oplus B_1}$$

b)

$$\mathbf{B_2 = G_2}$$

<u>B₁:</u>

G_0 \ G_2G_1	00	01	11	10
0	0	1	0	1
1	0	1	0	1

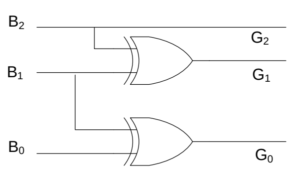
$$\mathbf{B_1} = \overline{G}_2G_1 + G_2\overline{G}_1 = \mathbf{G_1 \oplus G_2}$$

<u>B₀:</u>

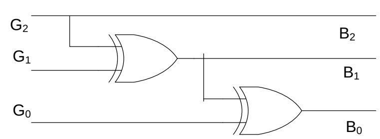
G_0 \ G_2G_1	00	01	11	10
0	0	1	0	1
1	1	0	1	0

$$\mathbf{B_0 = G_0 \oplus G_1 \oplus G_2}$$

a)

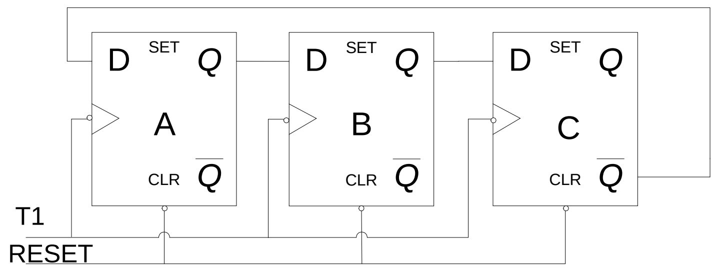


b.)



IV. REGISTRI I BROJAČI

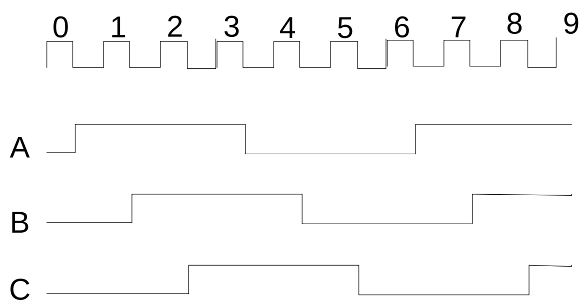
61. Pomerački registar od 3 bita vezan je kao na slici. Ako se on najpre resetuje, posle koliko takt impulsa će sadržaj registra ponovo biti **000** ?



D(t)	Q(t)	Q(t+!)
0	0	0
0	1	0
1	0	1
1	1	1

$D_A = \overline{C}$, $D_B = A$, $D_C = B$

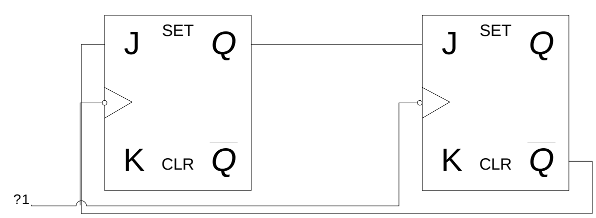
Rešenje:



Odgovor: Sadrzaj registra biće

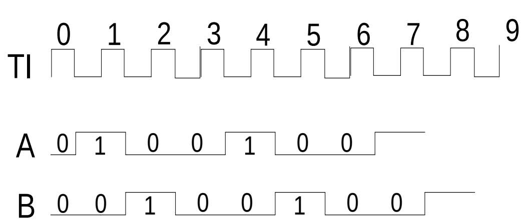
000 posle **6** impulsa

62. Sta predstavlja kolo na slici
a) brojač modula 3
b) pomerački registar od 3 bita
c) brojač modula 5
d)



Rešenje:

- kad je K=J=1, JK f-f radi kao T f-f
- vremenski dijagram stanja:



Ti	B	A	J	K	Qn+1
0	0	0	0	0	Qn
1	0	1	1	0	1
2	1	0	0	1	0
3	0	0	1	1	$\overline{Q}$

Odgovor: **a) brojač modula 3**

63. Projektovati sinhroni dekadni brojač u prirodnom **NBCD** kodu, koristeći **JK-MS** f-f-ove i elementarna logička kola.

Rešenje:

ul. imp.	t_n DCBA	t_{n+1} DCBA
0	0 0 0 0	0 0 0 1
1	0 0 0 1	0 0 1 0
2	0 0 1 0	0 0 1 1
3	0 0 1 1	0 1 0 0
4	0 1 0 0	0 1 0 1
5	0 1 0 1	0 1 1 0
6	0 1 1 0	0 1 1 1
7	0 1 1 1	1 0 0 0
8	1 0 0 0	1 0 0 1
9	1 0 0 1	0 0 0 0
10	0 0 0 0	

F-ja prelaza JK f-f-a: $\mathbf{Q_{n+1} = J\overline{Q}_n + \overline{K}Q_n}$

A:

BA \ DC	00	01	11	10
00	1	1	-	1
01	0	0	-	0
11	0	0	-	-
10	1	1	-	-

$A_{n+1} = \overline{A}_n = J\overline{A}_n + \overline{K}A_n$

$\mathbf{J_A = 1}$, $\overline{K} = 0$ $\mathbf{K_A = 1}$

B:

BA \ DC	00	01	11	10
00	0	0	-	0
01	1	1	-	0
11	0	0	-	-
10	1	1	-	-

$B_{n+1} = A\overline{D}\cdot\overline{B} + \overline{A}\cdot\mathbf{B}$

$\mathbf{J_B = A\overline{D}}$, $\overline{K} = \overline{A}$

$\mathbf{K_B = A}$

C:

BA \ DC	00	01	11	10
00	0	1	-	0
01	0	1	-	0
11	1	0	-	-
10	0	1	-	-

$C_{n+1} = \overline{B}C + AB\overline{C} + \overline{A}C$

$= AB\cdot\overline{C} + (\overline{A} + \overline{B})\,\mathbf{C}$

$\mathbf{J_C = AB}$, $\overline{K}_C = \overline{A} + \overline{B} = \overline{AB}$

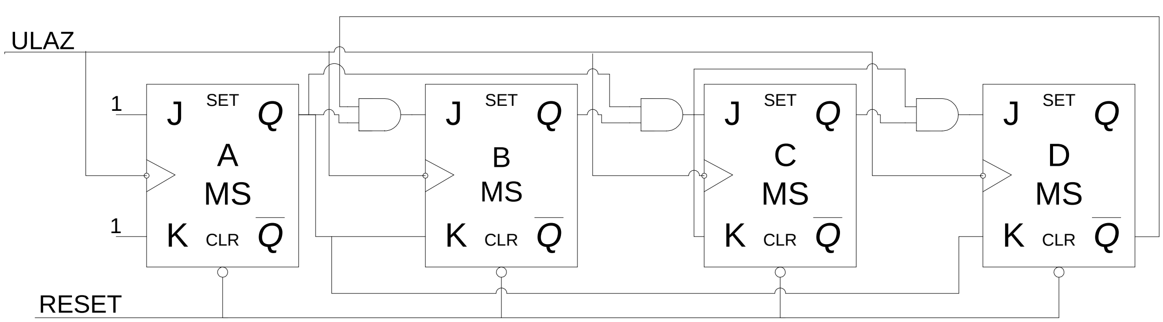
$\mathbf{K_C = AB}$

D:

BA \ DC	00	01	11	10
00	0	0	-	1
01	0	0	-	0
11	0	1	-	-
10	0	0	-	-

$D_{n+1} = ABC\cdot\overline{D} + \overline{A}\cdot D$

$\mathbf{J_D = ABC}$, $\overline{K}_D = \overline{A}$ $\mathbf{K_D = A}$



NAPOMENA: **<u>PRI MINIMIZACIJI VODITI RAČUNA DA SE U F-JI PRELAZA ZA ODGOVARAJUĆI IZLAZ (A,B,C,D), UVEK DOBIJE FORMA F-JE PRELAZA JK-FF-a.</u>**

64. Projektovati sinhroni brojač <u>sa modulom 10</u> za kod **2421 (Ajkenov)** koristeći **JK-T** f-f-ove i elementarna logička kola.

Rešenje:

ul. imp.	t_n DCBA	t_{n+1} DCBA
0	0 0 0 0	0 0 0 1
1	0 0 0 1	0 0 1 0
2	0 0 1 0	0 0 1 1
3	0 0 1 1	0 1 0 0
4	0 1 0 0	1 0 1 1
5	1 0 1 1	1 1 0 0
6	1 1 0 0	1 1 0 1
7	1 1 0 1	1 1 1 0
8	1 1 1 0	1 1 1 1
9	1 1 1 1	0 0 0 0
10	0 0 0 0	

$\mathbf{Q_{n+1}} = J\overline{Q} + \overline{K}Q$

A:

BA \ DC	00	01	11	10
00	1	1	1	-
01	0	-	0	-
11	0	-	0	0
10	1	-	1	-

$A_{n+1} = \overline{A}_n$

$\mathbf{J_A = 1}$, $\overline{K}_A = 0$, $\mathbf{K_A = 1}$

B:

BA \ DC	00	01	11	10
00	0	1	0	-
01	1	-	1	-
11	0	-	0	0
10	1	-	1	-

$B_{n+1} = A\overline{B} + \overline{A}B + C\overline{C}\overline{D} =$

$= (A + C\overline{D})\overline{B} + \overline{A}\cdot\mathbf{B}$

$\mathbf{J_B = A + C}\overline{D}$, $\overline{K}_B = \overline{A}$

$\mathbf{K_B = A}$

C:

BA \ DC	00	01	11	10
00	0	0	1	-
01	0	-	1	-
11	1	-	0	1
10	0	-	1	-

$C_{n+1} = \overline{C}AB + \overline{A}CD + \overline{B}CD$

$= AB\cdot\overline{C} + (\overline{A}D + \overline{B}D)\,\mathbf{C}$

$\mathbf{J_C = AB}$ $\quad \overline{K}_C = D(\overline{A} + \overline{B})$

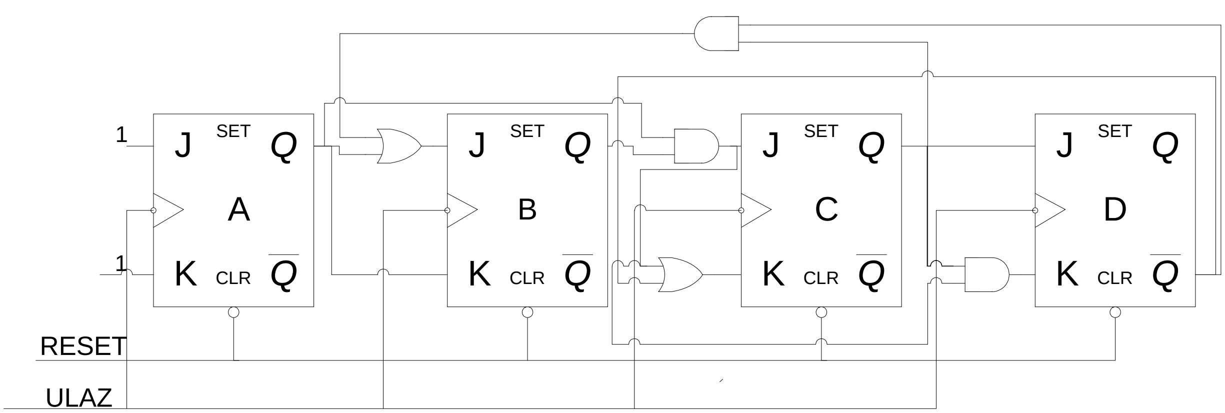
$\mathbf{K_C} = \overline{D} + \mathbf{AB}$

D:

BA \ DC	00	01	11	10
00	0	1	1	-
01	0	-	1	-
11	0	-	0	1
10	0	-	1	-

$D_{n+1} = C\overline{D} + \overline{B}D + \overline{A}D + \overline{C}D = C\cdot\overline{D} + (\overline{A} + \overline{B} + \overline{C})\cdot\mathbf{D}$

$\mathbf{J_D = C}$ $\quad \overline{K}_D = \overline{A} + \overline{B} + \overline{C}$ $\quad \mathbf{K_D = ABC}$



65. Projektovati sinhroni brojač do **10** za kod **"više 3" (BCDXS3**), koristeći JK f-f-ove i elementarna logička kola.

Rešenje:

ul. imp.	t_n DCBA	t_{n+1} DCBA
0	0 0 1 1	0 1 0 0
1	0 1 0 0	0 1 0 1
2	0 1 0 1	0 1 1 0
3	0 1 1 0	0 1 1 1
4	0 1 1 1	1 0 0 0
5	1 0 0 0	1 0 0 1
6	1 0 0 1	1 0 1 0
7	1 0 1 0	1 0 1 1
8	1 0 1 1	1 1 0 0
9	1 1 0 0	0 0 1 1
10	0 0 1 1	

A: Uporedjivanjem kolona A za t_n i t_{n+1}: $\mathbf{A_{n+1} = \bar{A}_n}$

$\mathbf{J_A = 1}$ $\mathbf{K_A = 1}$

B:

BA \ DC	00	01	11	10
00	-	0	1	0
01	-	1	-	1
11	0	0	-	0
10	-	1	-	1

$B_{n+1} = \bar{A}B + A\bar{B} + \bar{B}CD =$

$= (A+CD)\cdot\bar{B} + \bar{A}\cdot\mathbf{B}$

$\mathbf{J_B = A+CD}$ $\mathbf{K_B = A}$

C:

BA \ DC	00	01	11	10
00	-	1	0	0
01	-	1	-	0
11	1	0	-	1
10	-	1	-	0

$C_{n+1} = AB\bar{C} + \bar{B}C\bar{D} + \bar{A}C\bar{D} =$

$= AB\cdot\bar{C} + (\bar{A}\bar{D} + \bar{B}\bar{D})\,\mathbf{C}$

$\bar{K}_C = \bar{D}\cdot(\bar{A}+\bar{B})$

$\mathbf{J_C = AB}$

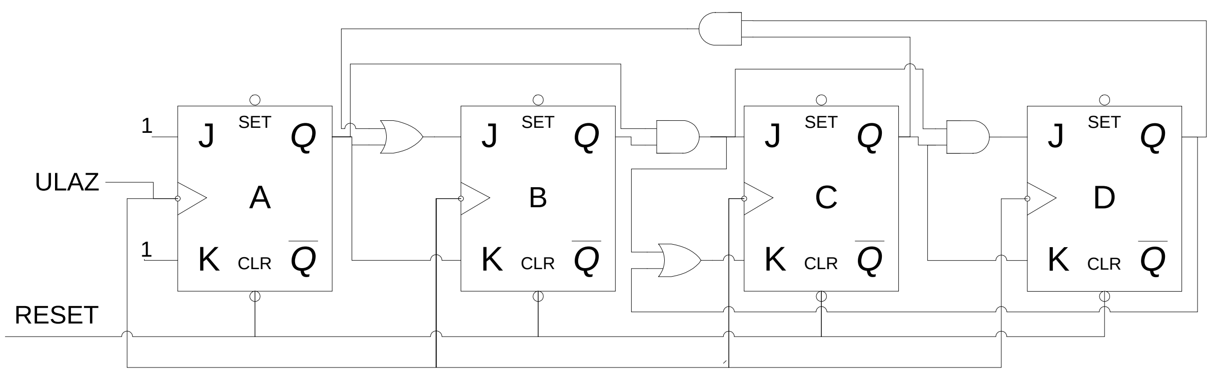
$\mathbf{K_C = AB + D}$

D:

BA \ DC	00	01	11	10
00	-	0	0	1
10	-	0	-	1
11	0	1	-	1
10	-	0	-	1

$D_{n+1} = \bar{C}D + ABC\bar{D} = ABC\cdot\bar{D} + \bar{C}\cdot\mathbf{D}$

$\mathbf{J_D = ABC}$ $\bar{K}_D = \bar{C}$ $\mathbf{K_D = C}$



66. Projektovati **sinhroni dekadni** brojač za **Grejov kod**, koristeći JK f-f-ove i elementarna logička kola.

Rešenje:

ul. imp.	t_n DCBA	t_{n+1} DCBA
0	0 0 0 0	0 0 0 1
1	0 0 0 1	0 0 1 1
2	0 0 1 1	0 0 1 0
3	0 0 1 0	0 1 1 0
4	0 1 1 0	0 1 1 1
5	0 1 1 1	0 1 0 1
6	0 1 0 1	0 1 0 0
7	0 1 0 0	1 1 0 0
8	1 1 0 0	1 1 0 1
9	1 1 0 1	0 0 0 0
10	0 0 0 0	

A:

BA \ DC	00	01	11	10
00	1	0	1	*
01	1	0	0	*
11	0	1	*	*
10	0	1	*	*

$$A_{n+1} = \bar{A}D + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + ABC + \bar{A}BC =$$
$$= \bar{A}(BC + \bar{B}\bar{C}) + A(BC + \bar{B}\bar{C}) + \bar{A}D =$$
$$= \bar{A}(\overline{B \oplus C}) + A(\overline{B \oplus C}) + \bar{A}D =$$
$$= (D + \overline{B \oplus C}) \cdot \bar{A} + (\overline{B \oplus C}) \cdot \mathbf{A}$$

$\mathbf{J_A = D +} \overline{B \oplus C}$ $\mathbf{K_A = B \oplus C}$

B:

BA \ DC	00	01	11	10
00	0	0	0	*
01	1	0	0	*
11	1	0	*	*
10	1	1	*	*

C:

BA \ DC	00	01	11	10
00	0	1	1	*
01	0	1	0	*
11	0	1	*	*
1	1	1	*	*

$$B_{n+1} = \bar{A}B + \bar{C}B + A\bar{C}\bar{B} =$$
$$= (\bar{A} + \bar{C})\,\mathbf{B} + A\bar{C} \cdot \bar{B}$$

$$C_{n+1} = C\bar{D} + \bar{A}B\bar{C} + \bar{A}C =$$
$$= (\bar{A} + \bar{D})\,\mathbf{C} + \bar{A}B \cdot \bar{C}$$

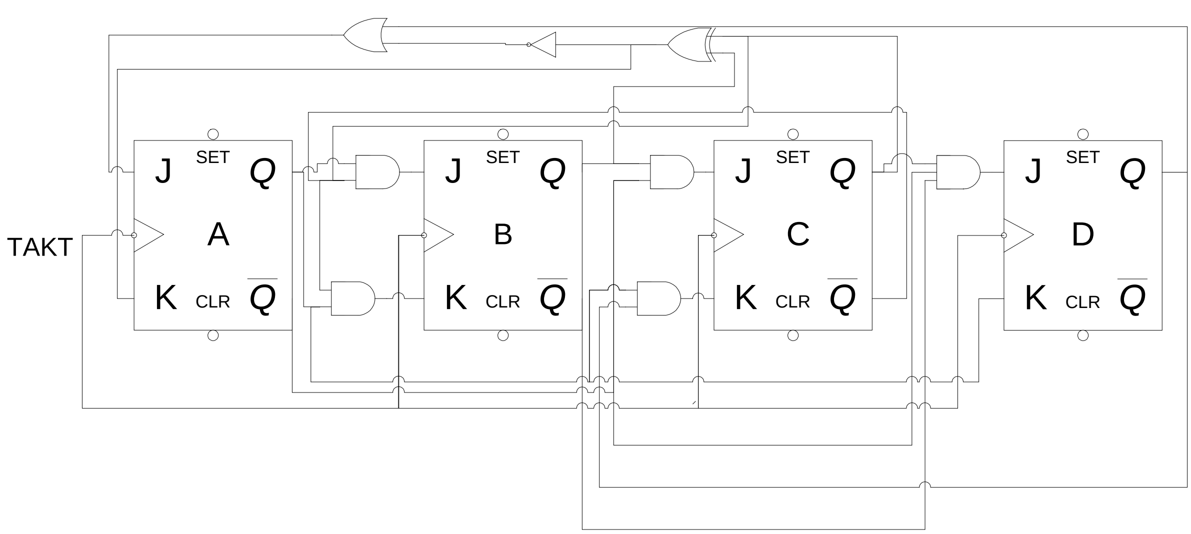
$\mathbf{J_B = A}\bar{C}$ **,** $\bar{K}_B = \bar{A} + \bar{C} = \overline{AC}$, $\mathbf{K_B = AC}$ $\mathbf{J_C =} \bar{A}\,\mathbf{B}$ **,** $\bar{K}_C = \bar{A} + \bar{D} = \overline{AD}$, $\mathbf{K_C = AD}$

D:

BA \ DC	00	01	11	10
00	0	1	1	*
01	0	0	0	*
11	0	0	*	*
10	0	0	*	*

$$D_{n+1} = \bar{A}\bar{B}C \cdot \bar{D} + \bar{A} \cdot \mathbf{D}$$

$\mathbf{J_D =} \bar{A}\bar{B}\,\mathbf{C}$ **,** $\bar{K}_D = \bar{A}$ $\mathbf{K_D = A}$



67. Projektovati **asinhroni dekadni brojač za kod "više 3"**, koristeći **JK** f-f-ove i univerzalna logička kola.

Rešenje:

ul. imp.	t_n DCBA	t_{n+1} DCBA	$T_D T_C T_B T_A$	$W_D W_C W_B W_A$
0	0 0 1 1	0 1 0 0	0 1 1 1	0 0 1 1
1	0 1 0 0	0 1 0 1	0 0 0 1	0 0 0 0
2	0 1 0 1	0 1 1 0	0 0 1 1	0 0 0 1
3	0 1 1 0	0 1 1 1	0 0 0 1	0 0 0 0
4	0 1 1 1	1 0 0 0	1 1 1 1	0 1 1 1
5	1 0 0 0	1 0 0 1	0 0 0 1	0 0 0 0
6	1 0 0 1	1 0 1 0	0 0 1 1	0 0 0 1
7	1 0 1 0	1 0 1 1	0 0 0 1	0 0 0 0
8	1 0 1 1	1 1 0 0	0 1 1 1	0 0 1 1
9	1 1 0 0	0 0 1 1	1 1 1 1	1 1 0 0
10	0 0 1 1			

T_A = T

T_B = (A+CD) T

T_B:

BA \ DC	00	01	11	10
00	*	0	1	0
01	*	1	*	1
11	1	1	*	1
10	*	0	*	0

T_C:

BA \ DC	00	01	11	10
00	*	0	1	0
01	*	0	*	0
11	1	1	*	1
10	*	0	*	0

T_D:

BA \ DC	00	01	11	10
00	*	0	1	0
01	*	0	*	0
11	0	1	*	0
10	*	0	*	0

W_A = A T

T_C = (AB+CD) T

T_D = (ABC+CD) T

W_B:

BA \ DC	00	01	11	10
00	*	0	0	0
01	*	0	*	0
11	1	1	*	1
10	*	0	*	0

W_C :

BA \ DC	00	01	11	10
00	*	0	1	0
01	*	0	*	0
11	0	1	*	0
10	*	0	*	0

W_D:

BA \ DC	00	01	11	10
00	*	0	1	0
01	*	0	*	0
11	0	0	*	0
10	*	0	*	0

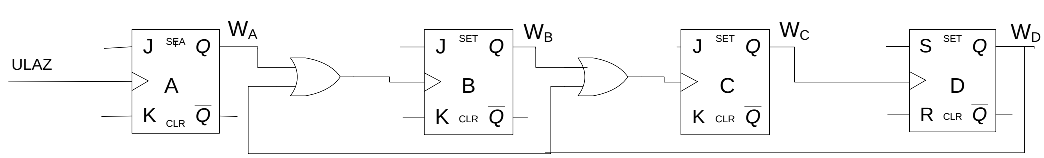
W_B = AB T

W_C = (ABC+CD) T

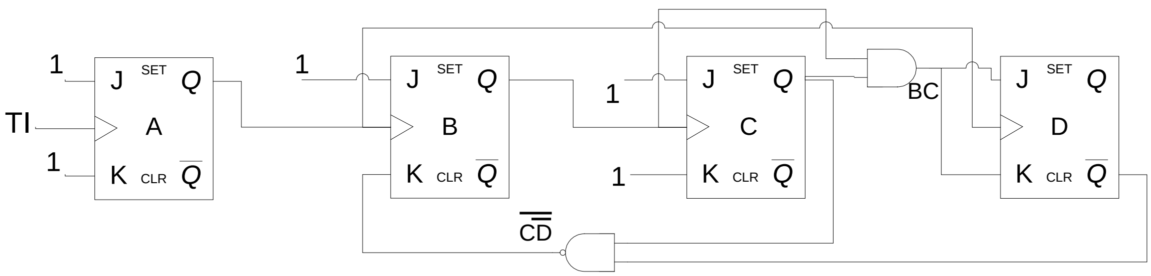
W_D = CD T

T – svaka promena stanja iz t_n u t_{n+1} daje log. vrednost 1

W – samo promena stanja sa 1 na 0 daje log. vrednost 1



68. Za asinhroni brojač sa slike odrediti osnovu brojanja i kod i nacrtati vremenski dijagram stanja na izlazima f-f-ova.

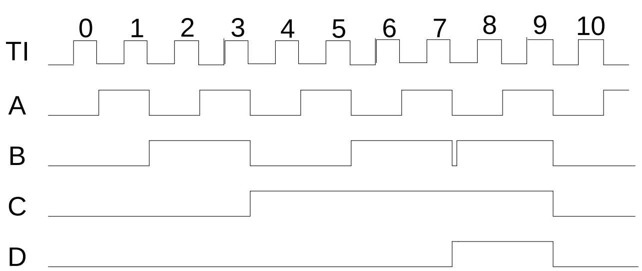


Rešenje:

$K_B = \overline{\overline{C}D} = \overline{C} + D$, $J_D = K_D = BC$

f-f **D** može da okida, tj. da radi kao **T** f-f kada je $\mathbf{J_D = K_D = 1}$ i kad f-f **A** menja svoje stabilno stanje sa **1** na **0.**

f-f **B** može da okida samo kada je $\mathbf{K_B = 1}$, jer kada je $\mathbf{K_B = 0}$, zbog $\mathbf{J_B = 1}$ njegov izlaz je stalno na nivou log. **1.**



-tabela stanja:

ul. imp.	DCBA
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
4	0 1 0 0
5	0 1 0 1
6	0 1 1 0
7	0 1 1 1
8	1 1 1 0
9	1 1 1 1
10	0 0 0 0

moduo -- 10

kod brojanja **2421**

69. Projektovati asinhroni dekadni brojač za kod **2421 (Ajkenov),** koristeći **JK** f-f-ove i elementarna logička kola.

Rešenje:

ul. imp.	t_n DCBA	t_{n+1} DCBA	$T_D T_C T_B T_A$	$W_D W_C W_B W_A$
0	0 0 0 0	0 0 0 1	0 0 0 1	0 0 0 0
1	0 0 0 1	0 0 1 0	0 0 1 1	0 0 0 1
2	0 0 1 0	0 0 1 1	0 0 0 1	0 0 0 0
3	0 0 1 1	0 1 0 0	0 1 1 1	0 0 1 1
4	0 1 0 0	1 0 1 1	1 1 1 1	0 1 0 0
5	1 0 1 1	1 1 0 0	0 1 1 1	0 0 1 1
6	1 1 0 0	1 1 0 1	0 0 0 1	0 0 0 0
7	1 1 0 1	1 1 1 0	0 0 1 1	0 0 0 1
8	1 1 1 0	1 1 1 1	0 0 0 1	0 0 0 0
9	1 1 1 1	0 0 0 0	1 1 1 1	1 1 1 1
10	0 0 0 0			

$T_A = T$

T_B:

BA \ DC	00	01	11	10
00	0	1	0	*
01	1	*	1	*
11	1	*	1	1
10	0	*	0	*

$T_B = (A+\bar{D}C)\,T$

T_C:

BA \ DC	00	01	11	10
00	0	1	0	*
01	0	*	0	*
11	1	*	1	1
10	0	*	0	*

T_D:

BA \ DC	00	01	11	10
00	0	1	0	*
01	0	*	0	*
11	0	*	1	0
10	0	*	0	*

$T_C = (AB+C\bar{D})\,T$ **$T_D = (C\bar{D}+ABC)\,T$**

$W_A = A\,T$

W_B:

BA \ DC	00	01	11	10
00	0	0	0	*
01	0	*	0	*
11	1	*	1	1
10	0	*	0	*

$W_C = T_D$

$W_C = (CD+ABC)\,T$

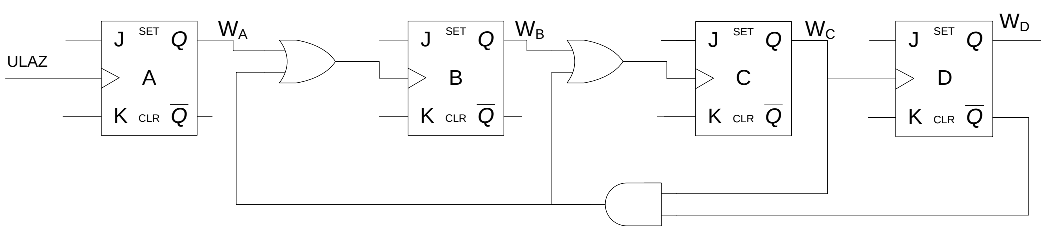
W_D:

BA \ DC	00	01	11	10
00	0	0	0	*
01	0	*	0	*
11	0	*	1	0
10	0	*	0	*

$T_A = T$

$T_B = W_A + C\bar{D}$

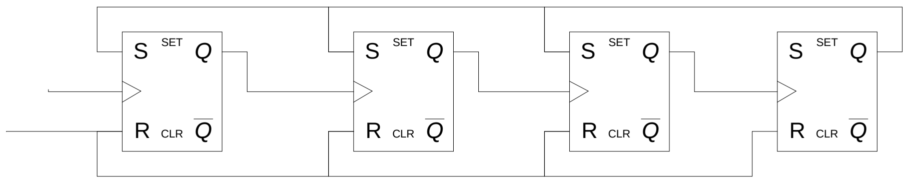
$W_B = AB\,T$ **$T_C = W_B + C\bar{D}$** **$T_D = W_C$** **$W_D = ABC\,T$**

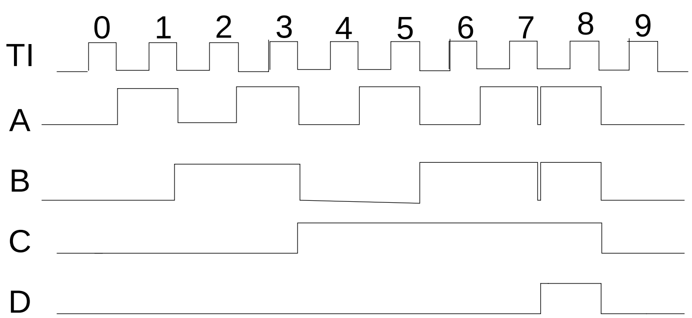


70. Konstruisati serijski (asinhroni) brojač **modula 9**. Nacrtati vremenski dijagram stanja na izlazima pojedinih f-f-ova i navesti zabranjena stanja brojača. Koristiti taktovane RS f-f-ove.

Rešenje:

ΔM = 2^4-9 = 16-9 =7   7= $2^0+2^1+2^2$ -- povratnu vezu treba dovesti na S ulaze svih prethodnih f-f-ova





- zabranjena stanja:.

1000	8
1001	9
1010	10
1011	11
1100	12
1101	13
1110	14

71. Konstruisati asinhroni brojač sa **osnovom 10** za prirodni binarno-kodirani decimalni **kod 8421** , koristeći **JK** f-f-ove i elementarna logička kola.

Rešenje:

ul. imp.	t_n DCBA	t_{n+1} DCBA	$T_D T_C T_B T_A$	W_D	W_C	W_B	W_A
0	0 0 0 0	0 0 0 1	0 0 0 1	0	0	0	0
1	0 0 0 1	0 0 1 0	0 0 1 1	0	0	0	1
2	0 0 1 0	0 0 1 1	0 0 0 1	0	0	0	0
3	0 0 1 1	0 1 0 0	0 1 1 1	0	0	1	1
4	0 1 0 0	0 1 0 1	0 0 0 1	0	0	0	0
5	0 1 0 1	0 1 1 0	0 0 1 1	0	0	0	1
6	0 1 1 0	0 1 1 1	0 0 0 1	0	0	0	0
7	0 1 1 1	1 0 0 0	1 1 1 1	0	1	1	1
8	1 0 0 0	1 0 0 1	0 0 0 1	0	0	0	0
9	1 0 0 1	0 0 0 0	1 0 0 1	1	0	0	1
10	0 0 0 0						

T_A = T

T_B:

BA \ DC	00	01	11	10
00	0	0	*	0
01	1	1	*	0
11	1	1	*	*
10	0	0	*	*

T_B = AD T

T_C:

BA \ DC	00	01	11	10
00	0	0	*	0
01	0	0	*	0
11	1	1	*	*
10	0	0	*	*

T_C = AB T

T_D:

BA \ DC	00	01	11	10
00	0	0	*	0
01	0	0	*	1
11	0	1	*	*
10	0	0	*	*

T_D =(AD+ABC) T

$\mathbf{W_A = A\,T}$ $\mathbf{W_B = AB\,T}$

$\mathbf{W_C = ABC\,T}$

$\mathbf{W_D = AD\,T}$

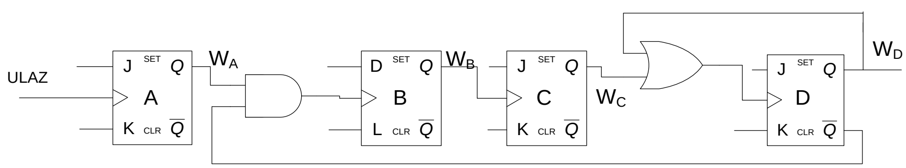
$\mathbf{W_D}$:

BA \ DC	00	01	11	10
00	0	0	*	0
01	0	0	*	1
11	0	0	*	*
10	0	0	*	*

$\mathbf{W_C}$:

BA \ DC	00	01	11	10
00	0	0	0	*
01	0	*	0	*
11	0	*	1	0
10	0	*	0	*

$\mathbf{T_A = T}$ $\mathbf{T_B = W_A}\overline{D}$ $\mathbf{T_C = W_B}$ $\mathbf{T_D = W_D + W_C}$



72. Projektovati sinhroni brojač **modula 7** koji će da broji u **Grejovom kodu.** Koristiti **JK-MS** f-f-ove i elementarna logička kola.

Rešenje:

db	t_n CBA	t_{n+1} CBA
0	0 0 0	0 0 1
1	0 0 1	0 1 1
2	0 1 1	0 1 0
3	0 1 0	1 1 0
4	1 1 0	1 1 1
5	1 1 1	1 0 1
6	1 0 1	0 0 0
7	0 0 0	

A:

A \ CB	00	01	11	10
0	1	0	1	*
1	1	0	1	0

B:

A \ CB	00	01	11	10
0	0	1	1	*
1	1	1	0	0

C:

A \ CB	00	01	11	10
0	0	1	1	*
1	0	0	1	0

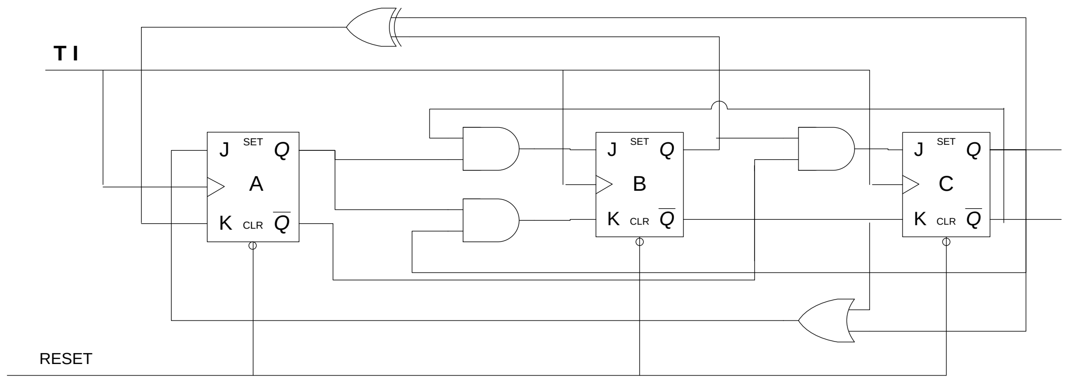
$\mathbf{A_{n+1}} = \overline{A}\,\overline{B} + \overline{A}C + A\overline{B}\,\overline{C} + ABC = (\overline{B} + C)\overline{A} + (\overline{B}\,\overline{C} + BC)A =$

$= (\overline{B} + C)\cdot\overline{A} + (\overline{B \oplus C})\,\mathbf{A}$ $\mathbf{J_A} = \overline{B}\,\mathbf{+C}$ $\mathbf{K_A = B \oplus C}$

$\mathbf{B_{n+1}} = \overline{A}B + \overline{C}B + A\overline{C}\,\overline{B} = A\overline{C}\,\overline{B} + (\overline{A} + \overline{C})B = A\overline{C}\cdot\overline{B} + \overline{AC}\cdot\,\mathbf{B}$

$\mathbf{J_B = A}\overline{C}$ $\mathbf{K_B = AC}$

$\mathbf{C_{n+1}} = \overline{A}B\cdot\overline{C} + B\,\mathbf{C}$ $\mathbf{J_C} = \overline{A}\,\mathbf{B}$ $\mathbf{K_C = B}$



73. Projektovati sinhroni brojač iz zad.66. sa **modulom 5.**

Rešenje:

db	t_n CBA	t_{n+1} CBA
0	0 0 0	0 0 1
1	0 0 1	0 1 1
2	0 1 1	0 1 0
3	0 1 0	1 1 0
4	1 1 0	0 0 0
5	0 0 0	

A:

A \ CB	00	01	11	10
0	1	0	0	*
1	1	0	*	*

B:

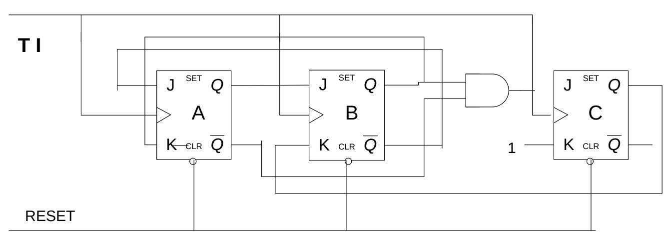
A \ CB	00	01	11	10
0	0	1	0	*
1	1	1	*	*

C:

A \ CB	00	01	11	10
0	0	1	0	*
1	0	0	*	*

$\mathbf{A_{n+1}} = \overline{B}\cdot\overline{A}+\overline{B}\cdot\mathbf{A}$ $\mathbf{B_{n+1}} = A\cdot\overline{B}+\overline{C}\cdot\mathbf{B}$ $\mathbf{C_{n+1}} = \overline{A}B\cdot\overline{C}$

$\mathbf{J_A = \overline{B}}$ $\mathbf{K_A = B}$ $\mathbf{J_B = A}$ $\mathbf{K_B = C}$ $\mathbf{J_C = \overline{A}B}$ $\mathbf{K_C = 1}$



74. Projektovati **sinhroni** brojač u **Grejovom kodu modula 6**, koristeći **JK-MS** f-f-ove.

Rešenje:

ul. imp	t_n CBA	t_{n+1} CBA
0	0 0 0	0 0 1
1	0 0 1	0 1 1
2	0 1 1	0 1 0
3	0 1 0	1 1 0
4	1 1 0	1 1 1
5	1 1 1	0 0 0
6	0 0 0	

A:

A \ CB	00	01	11	10
0	1	0	1	*
1	1	0	0	*

B:

A \ CB	00	01	11	10
0	0	1	1	*
1	1	1	0	*

C:

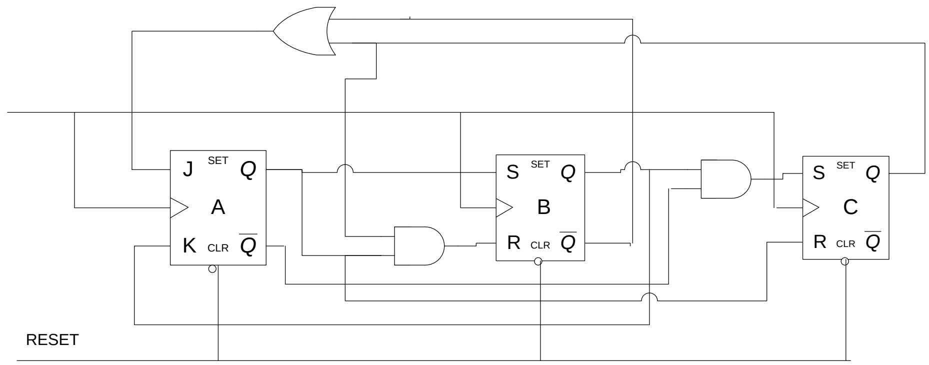
A \ CB	00	01	11	10
0	0	1	1	*
1	0	0	0	*

$\mathbf{A_{n+1}} = \overline{B}\,\overline{A}+\overline{B}A+C\overline{A} = (\overline{B}+\mathbf{C})\cdot\overline{A}+\overline{B}\,\mathbf{A}$

$\mathbf{B_{n+1}} = \overline{A}B+A\overline{B}+\overline{C}B = \mathbf{A}\cdot\overline{B} + (\overline{A}+\overline{C})\,\mathbf{B}$

$\mathbf{C_{n+1}} = \overline{A}B\cdot\overline{C}+\overline{A}\cdot C$

$\mathbf{J_A = \overline{B}+C}$ $\mathbf{K_A = B}$ $\mathbf{J_B = A}$ $\mathbf{K_B = AC}$ $\mathbf{J_C = \overline{A}B}$ $\mathbf{K_C = A}$



75. Projektovati paralelni (sinhroni) brojač **modula 6** koji će da broji u **Ajkenovom kodu**. Koristiti **JK – MS** f-f-ove i elementarna logička kola.

Rešenje:

CLK	t_n D C B A	t_{n+1} D C B A
0	0 0 0 0	0 0 0 1
1	0 0 0 1	0 0 1 0
2	0 0 1 0	0 0 1 1
3	0 0 1 1	0 1 0 0
4	0 1 0 0	1 0 1 1
5	1 0 1 1	0 0 0 0
6	0 0 0 0	

$A_{n+1} = \overline{A_n}$

$J_A = 1$, $K_A = 1$

B :

BA \ DC	00	01	11	10
00	0	1	*	*
01	1	*	*	*
11	0	*	*	0
10	1	*	*	*

$$B_{n+1} = \overline{A}B + A\overline{B} + C\overline{B} =$$
$$= (A + C)\cdot\overline{B} + \overline{A}\cdot B$$

$J_B = A + C$

$K_B = A$

C:

BA \ DC	00	01	11	10
00	0	0	*	*
01	0	*	*	*
11	1	*	*	0
10	0	*	*	*

D:

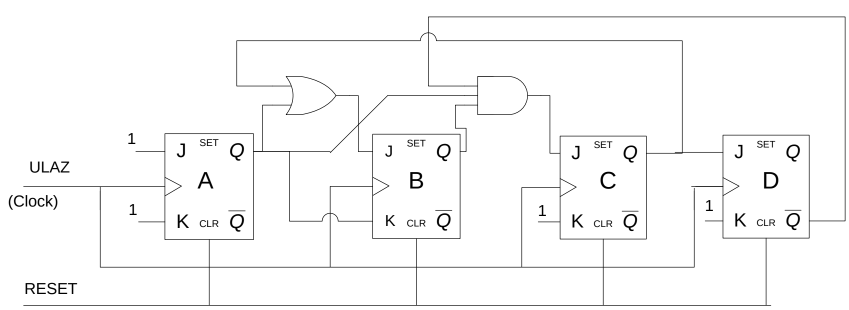
BA \ DC	00	01	11	10
00	0	1	*	*
01	0	*	*	*
11	0	*	*	0
10	0	*	*	*

$$C_{n+1} = AB\overline{D}\cdot\overline{C}$$

$J_C = AB\overline{D}$, $K_C = 1$

$$D_{n+1} = C\cdot\overline{D}$$

$J_D = C$, $K_D = 1$

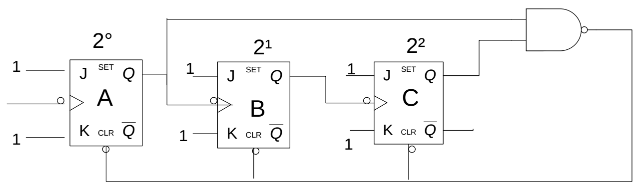


76. Projektovati brojač **modula 5** koristeći **JK-MS** f-f-ove sa asinhronim reset ulazima koji će da broji u prirodnom kodu. Kako se od ovog brojača može dobiti dekadni brojač?

Rešenje:

Brojač treba da broji tj. da zauzima stanja od 0 do 4, a da preskače stanja 5,6 i 7. Dakle, posle 5-og stanja treba da se vrati na početno stanje.

$5 = 2^0+2^2$ -- sa ovih pozicija treba dovesti izlaze na NI kolo i dobijeni impuls iskoristiti za resetovanje brojača



Dekadni brojač se moze dobiti ako se ispred f-f-a A doda još jedan f-f .

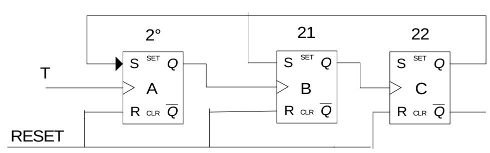
77. Projektovati asinhroni brojač **modula 5** koji će da broji u prirodnom kodu, koristeći **RST** f-f-ove.

Koji brojač se dobija ako se ispred f-f-a A doda još jedan f-f?

Rešenje:

n = 3 (broj f-f-ova) $M_0 = 2^3 = 8$ M = 5 $\Delta M = M_0 - M = 8 - 5 = 3$

$3 = 2^0 + 2^1$ -- povratnu vezu treba vratiti sa zadnjeg, na f-f-ove sa pozicionim vrednostma 2^0 i 2^1



78. Konstruisati kružni brojač modula **n = 4** sa **JK-MS** f- f-ovima i dati tabelu prelaza.

a) Kako od postojećeg brojača dobiti Džonsonov brojač **modula 8 i modula 7** ?

b) Kako od datog brojača dobiti **pomerački** a kako **kružni registar od 4 bita ?**

Rešenje:

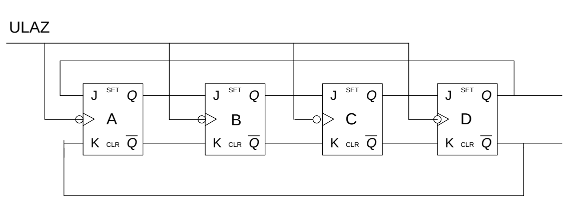


Tabela stanja za kružni brojač:

TI	A	B	C	D
0	1	0	0	0
1	0	1	0	0
2	0	0	1	0
3	0	0	0	1
4	1	0	0	0

Tabela stanja za Džonsonov brojač:

TI	A	B	C	D
0	0	0	0	0
1	1	0	0	0
2	1	1	0	0
3	1	1	1	0
4	1	1	1	1
5	0	1	1	1
6	0	0	1	1
7	0	0	0	1
8	0	0	0	0

a) Džonsonov brojač modula 8 dobiće se ako se ukrste povratne veze sa zadnjeg na prvi f-f, a modula 7 ako se jedna od ukrštenih veza pomeri na prethodni f-f. Takođe treba docrtati asinhrone RESET ulaze na svim f-f-ovima radi dovođenja brojača u početno stanje.

b) Za dobijanje pomeračkog registra treba na ulazu prvog f-f-a dodati invertor između J i K ulaza, da bi on radio kao D f-f i ukloniti povratne veze. Za kružni registar treba ostaviti povratnu vezu između izlaza D i ulaza J prvog f-f-a.

V. ARITMETIČKA KOLA

79. Dat je broj u binarnom brojnom sistemu **N = -10101**. Naći dvojični i jedinični komplement.

Rešenje:

- broj cifara: n = 5 i jos jedna cifra za algebarski znak: n +1 = 6.
- za konstantu Q usvajamo: $Q = 2^6$

Dvojični komplement (komplement dvojke) je:

$\overline{N}_2 = Q - |N| = 1000000 - 10101 =$ **101011**

```
 1000000
-  10101
 -------
  101011
```

Jedinični komplement je:

$\overline{N}_1 = 2^6 - 1 - 10101 = 111111 - 10101 =$ **101010**

jer je $Q = 2^6 - 1 = 1000000 - 1 = 111111$.

Dvojični komplement se može dobiti iz jediničnog, tako sto se cifri najmanje tezine doda 1.
Jedinični komplement se dobija tako što se svaka cifra binarnog broja jedinično komplementira.
Negativni brojevi na mestu najveće tezine, posle komplementiranja imaju cifru 1 (za algebarski znak).

Dvojični komplement se moze dobiti i tako sto se sve cifre datog binarnog broja prepišu s desna na levo do prve jedinice (uključujuci i nju) a sve ostala jedinično komplementiraju.

80. Naći dvojični i jedinični komplement broja: **N = -1101,11.**

Rešenje:

-- jedinični komplement je: $\overline{N}_1$ **= 10010,00** a dvojični: $\overline{N}_2$ **= 10010,01**

81. Naći jedinični i dvojični komplement broja: a) **N = -110100**. b) **N = -101,110**

Rešenje:

a) jedinični: $\overline{N}_1$ **= 1001011** dvojični: $\overline{N}_2 = 1001011 + 1 =$ **1001100** direktnim primenom pravila dvojičnog komplementa dobićemo isti broj:

$\bar{N}_2$ **= 1001100** --s desna ulevo prepisane su tri cifre (do prve jedinice, uključujući i nju) a ostale jedinično komplementirane

b) $\bar{N}_1$ **= 1010,001** $\bar{N}_2$ **= 1010,010**

82. Realizovati paralelni komplementor dvojke za prirodni binarni kod, koristeći elementarna logička kola.

Rešenje:

d.b. (N)	kod 8421 D	C	B	A	kom. dv. D′	C′	B′	A′	d.b ($\bar{N}$)
0	0	0	0	0	0	0	0	0	0
1	0	0	0	1	1	1	1	1	15
2	0	0	1	0	1	1	1	0	14
3	0	0	1	1	1	1	0	1	13
4	0	1	0	0	1	1	0	0	12
5	0	1	0	1	1	0	1	1	11
6	0	1	1	0	1	0	1	0	10
7	0	1	1	1	1	0	0	1	9
8	1	0	0	0	1	0	0	0	8
9	1	0	0	1	0	1	1	1	7
10	1	0	1	0	0	1	1	0	6
11	1	0	1	1	0	1	0	1	5
12	1	1	0	0	0	1	0	0	4
13	1	1	0	1	0	0	1	1	3
14	1	1	1	0	0	0	1	0	2
15	1	1	1	1	0	0	0	1	1

A′ = A

B:

BA \ DC	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	0	0	0	0
10	1	1	1	1

C:

BA \ DC	00	01	11	10
00	0	1	1	0
01	1	0	0	1
11	1	0	0	1
10	1	0	0	1

$B = A\bar{B} + \bar{A}B$

$\mathbf{B' = A \oplus B}$

C′ = AC+ BC+ABC = C (A+B) + C AB

F = A+B $\quad \bar{F} = \overline{A+B} = \bar{A}\cdot\bar{B}$

$C' = \bar{C}F + C\bar{F} = C \oplus F \qquad \mathbf{C' = C \oplus (A+B)}$

D:

BA \ DC	00	01	11	10
00	0	1	0	1
01	1	1	0	0
11	1	1	0	0
10	1	1	0	0

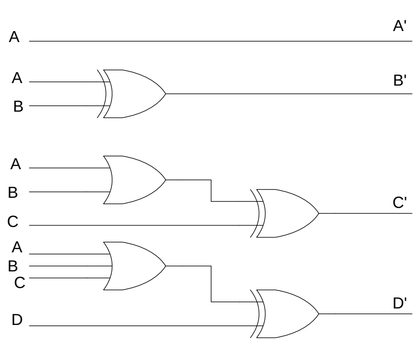
$D' = C\bar{D} + B\bar{D} + A\bar{D} + \bar{A}\bar{B}\bar{C}D$

$D' = \bar{D}\cdot(A+B+C) + D\cdot\bar{A}\bar{B}\bar{C}$

F = A+B+C

$D' = \bar{D}F + D\bar{F} = D \oplus F$

$\mathbf{D' = D \oplus (A+B+C)}$



83. Realizovati paralelni komplementor dvojke za prirodni BCD kod, kristeći:
a) logička NI kola b) elementarna logička kola

Rešenje:

db.	NBCD DCBA	komp. dv. D′C′B′A′
0	0 0 0 0	0 0 0 0
1	0 0 0 1	1 1 1 1
2	0 0 1 0	1 1 1 0
3	0 0 1 1	1 1 0 1
4	0 1 0 0	1 1 0 0
5	0 1 0 1	1 0 1 1
6	0 1 1 0	1 0 1 0
7	0 1 1 1	1 0 0 1
8	1 0 0 0	1 0 0 0
9	1 0 0 1	0 1 1 1

a) **A′ = A**

B′:

BA \ DC	00	01	11	10
00	0	0	*	0
01	1	1	*	1
11	0	0	*	*
10	1	1	*	*

$B' = (A+B)\,(\overline{A}+\overline{B})$

$\mathbf{B'} = \overline{\overline{A}\,\overline{B}} \cdot \overline{AB}$

C′:

BA \ DC	00	01	11	10
00	0	1	*	0
01	1	0	*	1
11	1	0	*	1
10	1	0	*	*

D′:

BA \ DC	00	01	11	10
00	0	1	*	1
01	1	1	*	0
11	1	1	*	*
10	1	1	*	*

$C' = (\overline{A}+\overline{C})(\overline{B}+\overline{C})\,(A+B+C)$

$\mathbf{C'} = \overline{AC} \cdot \overline{BC} \cdot \overline{A}\,\overline{B}\,\overline{C}$

$D' = (\overline{A}+\overline{D})\,(A+B+C+D)$

$\mathbf{D'} = \overline{AD} \cdot \overline{\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}}$

b) **A′ = A**

B′:

BA \ DC	00	01	11	10
00	0	0	*	0
01	1	1	*	1
11	0	0	*	*
10	1	1	*	*

C′:

BA \ DC	00	01	11	10
00	0	1	*	0
01	1	0	*	1
11	1	0	*	*
10	1	0	*	*

$\mathbf{B'} = A\overline{B} + \overline{A}B = A \oplus B$

$C' = A\overline{C} + B\overline{C} + \overline{A}\,\overline{B}C = \overline{C}(A+B) + \overline{A}\,\overline{B}C =$

$= \overline{C}(A+B) + \overline{(A+B)}C$

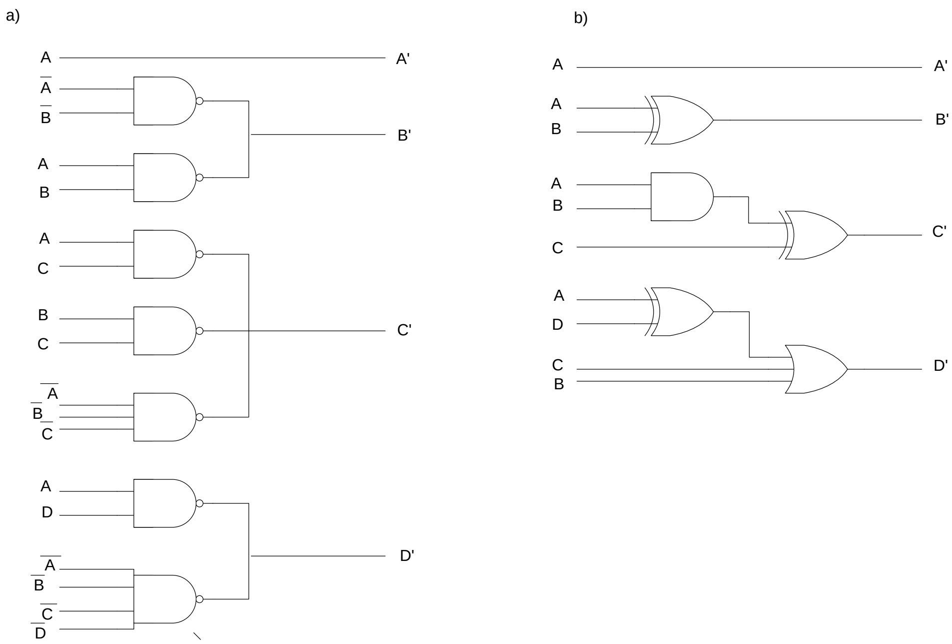
$\mathbf{C' = C \oplus (A+B)}$

D:

BA \ DC	00	01	11	10
00	0	1	*	1
01	1	1	*	0
11	1	1	*	*
10	1	1	*	*

$D' = B + C + A\overline{D} + \overline{A}D$

$\mathbf{D' = B + C + A \oplus D}$



84. Prikazati proces deljenja brojeva **19 i 6** na delitelju.

Rešenje:

$19_{10} = 10011_2$ $6_{10} = 110_2$

10011 : 110 = 11 – kolicnik $\overline{X}' = 1001$ kompl. jedinice

- 110 $\overline{X}'' = \mathbf{1010}$ kompl. dvojke broja 6

00111

- 110

001 -- ostatak

-- na racunaru:

pomeranje 1. taktom 00010011 : 0110 = **0 0 0 1 1**

1010

zabrana upisa u R 1011 upis u Q

pomeranje 2. taktom 00010

1010

zabrana upisa u R 1100 upis u Q

pomeranje 3. taktom 00100

1010

zabrana upisa u R 1110 upis u Q

pomeranje 4. taktom 01001

1010

upis u R, pomer. 5. takt.1 00111 upis u Q

1010

upis u R, ostatak 1 **0001** upis u Q

85. Prikazati proces deljenja brojeva **22 i 7** na delitelju.

Rešenje:

$22_{10} = 10110_2$ – deljenik   7 = 111 – delilac

10110 : 111 = **11**   N = 0111 $\overline{N}''$= 1001 – kompl. dvojke broja 7
-111   količnik
1000
- 111
001 -- ostatak

na delitelju:

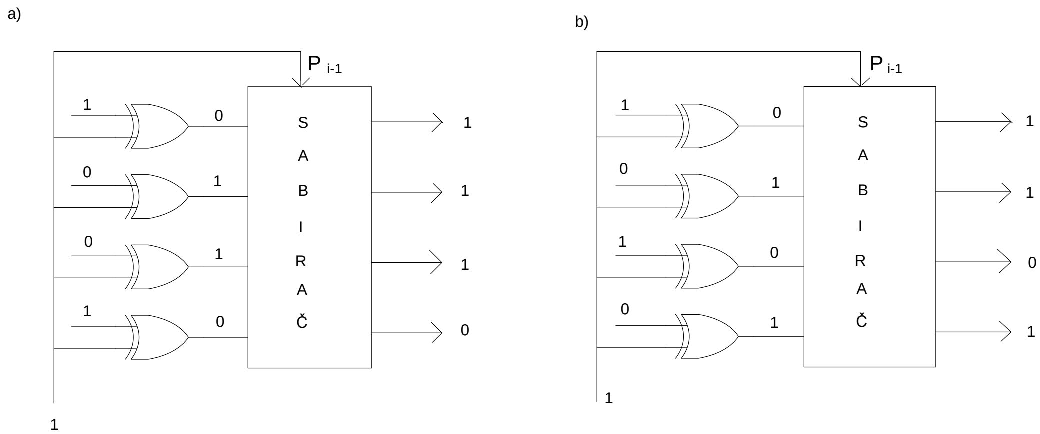
00010110 : 0111 = **0 0 0 1 1**   količnik (u Q)
1001
1. zabrana upisa u R 1010   upis u Q
2. 00010
1001
zabrana upisa u R   1011   upis u Q
3. 00101
1001
zabrana upisa u R   1110   upis u Q
4. 01011
1001
upis u R   10100   upis uQ
5. 01000
1001
upis u R   1**0001**   ostatak

86. Prikazati proces deljenja brojeva **12 i 4** na delitelju.
Rešenje:

-- delilac: $4_{10} = 100_2$ N = 0100 $\overline{N}''$= 1100

1. 0001100 : 0100 = **0 0 1 1**   upis u Q
1100
1101
2. 00011
1100
1111
3. 00110
1100
1 0010
4. 00100
1100
1 **0000**   ostatak u R

87. Naći stanja na izlazima **EKS-ILI** kola i na izlazima sabirača, ako je stanje na ulazima:
a) **1 0 0 1** b) **0 1 0 1**



Rešenje: **a) 0 1 1 1** **b) 1 0 1 1**

EKS-ILI kola jedinično komplementiraju ulaz a sabirač dodaje **1** i na izlazu se dobija komplement dvojke sa ulaza EKS-ILI kola.

88. Koliko takt-intervala je potrebno za izvodjenje mnozenja brojeva **58 i 37** pomoću
a) akumulacionog množača
b) redno-paralelnog množača

Rešenje:

A·B = $58_{10} \cdot 37_{10} = 111010_2 \cdot 100101_2$

Odgovor: **a)** potrebno je **37** takt-intervala (broj B)
b) potrebno je **6** taktnih intervala (broj cifara broja B)